King Fahd University of Petroleum & Minerals  
Department of Mathematics & Statistics  
Math 302 Major Exam I  
The Summer Semester of 2009-2010 (093)  
Time Allowed: 120 Minutes

Name: ___________________________ ID#: __________________
Section/Instructor: _______________ Serial #: _______________

- Mobiles and calculators are not allowed in this exam.
- Write all steps clear.

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Q:1 Let $S_a = \{(x, y, ax, a^2 - 5a + 6) \mid x, y, a \in \mathbb{R}\}$ be a subset of $\mathbb{R}^4$.

(a) (3 points) Find all values of $a$ for which $S_a$ is a subspace of $\mathbb{R}^4$.

(b) (7 points) For each value of $a$ obtained in part (a), find a basis for $S_a$.
Q:2 (10 points) Find the general solution of the linear system

\[
\begin{align*}
3x - 2y + z &= 6 \\
x + 10y - z &= 2 \\
-3x - 2y + z &= 0
\end{align*}
\]
Q:3 Let \( A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 0 & 2 \\ 0 & 0 & 3 \end{bmatrix} \).

(a) (7 points) Find a matrix \( P \) that diagonalizes \( A \). Also write \( P^{-1}AP \).

(b) (5 points) Find an orthogonal matrix \( Q \) that also diagonalizes \( A \).
Q:4 (10 points) Find equation of the tangent plane and normal line to the surface $2x - \cos(xyz) = 2$ at $\left(1, \frac{\pi}{2}, 1\right)$
Q: 5 Let \( \mathbf{F}(x, y, z) = e^{xyz} \mathbf{i} + \ln(xyz) \mathbf{j} + \tan^{-1}(xyz) \mathbf{k} \).

(a) (5 points) Find \( \nabla \cdot \mathbf{F} \)

(b) (6 points) Find \( \nabla \times \mathbf{F} \).
Q:6 (10 points) Evaluate the integral \( \int_C (2x + 3y) \, ds \), where the curve \( C \) is given by \( x = y = \sqrt{z} \) for \( 0 \leq y \leq 2 \).
Q:7 (12 points) Let $F(x, y) = 2xy \mathbf{i} + 3x^2 \mathbf{j}$ and $C$ is the boundary of the region determined by the graphs of $x = 0$, $x^2 + y^2 = 1$, $x \geq 0$. Use Greens’ theorem to evaluate $\int_C F \cdot d\mathbf{R}$. 