Name ? ID 9.5 Q5

Q1

If $f(x) = x\cos^{-1}(2x) - \frac{1}{2}\sqrt{1-4x^2}$, then $f'(x) =$

$f'(x) = \cos^{-1}(2x) - \frac{2x}{\sqrt{1-4x^2}} - \frac{1}{4}\,\frac{-8x}{\sqrt{1-4x^2}}$

(a) $4\cos^{-1}(2x)$

(b) $\cos^{-1}(2x)$

$= \cos^{-1}(2x) - \frac{2x}{\sqrt{1-4x^2}} + \frac{-2x}{\sqrt{1-4x^2}}$

(c) $\frac{1}{4}\cos^{-1}(2x)$

$f'(x) = \cos^{-1}(2x) + \frac{2x + 2x}{\sqrt{1-4x^2}}$

(d) $\cos^{-1}(2x) - \frac{x}{\sqrt{1-4x^2}}$

$f'(x) = \cos^{-1}(2x)$

(e) $\frac{\cos^{-1}(2x)}{\sqrt{1-4x^2}}$

Q2

If $1 + xy + y\cos y = e^{1-x} - \frac{\pi}{2}$,

then y' at $\left(1, -\frac{\pi}{2}\right)$ is equal

(a) $\frac{\pi}{2}$

$y' \Rightarrow 0 + y + xy' + y'\cos y - yy'\sin y = -e^{1-x} - 0$

(b) -1

$xy' + y'\cos y - yy'\sin y = -e^{1-x} - y$

(c) 0

(d) -2

$y' = \frac{-e^{1-x} - y}{(x + \cos y - y\sin y)}$

(e) 1

at $\left(1, -\frac{\pi}{2}\right) \Rightarrow y' = \frac{-1 + \frac{\pi}{2}}{\left(1 + 0 + \frac{\pi}{2}\sin -\frac{\pi}{2}\right)}$

$= \frac{\frac{-2+\pi}{2}}{\left(1 - \frac{\pi}{2}\right)}$

$= \frac{-1 + \frac{\pi}{2}}{-\left(-1 + \frac{\pi}{2}\right)}$

$= -1$

Q3

If $y = x^{\tan x}$, then $y'\left(\dfrac{\pi}{4}\right) =$

(a) 1

(b) $\dfrac{\pi}{4}\ln\dfrac{\pi}{4}$

(c) $1 + \dfrac{\pi}{4}\ln\dfrac{\pi}{4}$

(d) $\dfrac{\pi}{2}\ln\dfrac{\pi}{4}$

(e) $1 + \dfrac{\pi}{2}\ln\dfrac{\pi}{4}$

$\ln y = \tan x \ln x$

$\dfrac{y'}{y} = \sec^2 x \ln x + \dfrac{\tan x}{x}$

$y' = x^{\tan x}\left(\sec^2 x \ln x + \dfrac{\tan x}{x}\right)$

$y'\left(\dfrac{\pi}{4}\right) = \dfrac{\pi}{4}\left(\left(\dfrac{1}{\frac{1}{\sqrt{2}}}\right)^2 \ln\dfrac{\pi}{4} + \dfrac{1}{\frac{\pi}{4}}\right)$

$= \dfrac{\pi}{4}\left(2\ln\dfrac{\pi}{4} + \dfrac{4}{\pi}\right)$

$= 2\,\dfrac{\pi}{4}\ln\dfrac{\pi}{4} + 1$

$= \dfrac{\pi}{2}\ln\dfrac{\pi}{4} + 1$

Q4

If $y = 3^x \cdot x^3$, then $y'(1) =$

(a) 6

(b) $9 + 3\ln 3$

(c) 12

(d) $9 + \ln 9$

(e) $3 + 3\ln 3$

$\ln y = x\ln 3 + 3\ln x$

$\dfrac{y'}{y} = \ln 3 + 0 + \dfrac{3}{x}$

$\dfrac{y'}{y} = \ln 3 + \dfrac{3}{x}$

$y' = (3^x \cdot x^3)\left(\ln 3 + \dfrac{3}{x}\right)$

$y'(1) = (3\cdot 1)(\ln 3 + 3)$

$y'(1) = 3\ln 3 + 9$

Q5

An equation of the tangent line to the curve $xe^y = y - 1$ at $x = 0$ is given by

$y(0) \Rightarrow 0 = y - 1 \Rightarrow y = 1$

$y' \Rightarrow e^y + xy'e^y = y' - 0$

$\cancel{xy'e^y}$

$y' - xy'e^y = e^y$

$y' = \dfrac{e^y}{(1 - xe^y)}$

$y'(0) = \dfrac{e^y}{(1 - 0)}$

$y'(0) = e^1$

(a) $y = e \cdot x$

(b) $y = x + 1$

(c) $y = e \cdot x + 1$

(d) $y = 2e \cdot x + 1$

(e) $y = x$

$y - 1 = e^y(x - 0)$

$y = e^y x + 1$

$y = ex + 1$

Q6

Suppose that $F(x) = f(g(x))$ and $g(3) = 6$, $g'(3) = 4$, $f'(3) = 2$ and $f'(6) = 7$. Then $F'(3)$ is equal to

$F'(3) = f'(g(3)) \cdot g'(3)$

$= f'(6) \cdot (4)$

$= 7(4)$

$= 28$

(a) 8

(b) 14

(c) 24

(d) 42

(e) 28

$\dfrac{\cancel{40}x + 12}{\sqrt{20x^2 + 12x + 1}}$

$\cancel{+12}\ 1\cancel{32}$

[redacted]

Q7

$(2e^x - x)^2$

$u^2 = 4e^{2x} - 4xe^x + x^2$

If $y = \sin(u^2 - 4)$ and $u = 2e^x - x$, then $\left.\dfrac{dy}{dx}\right|_{x=0} =$

(a) 4

(b) −4

(c) −2

(d) 0

(e) 2

$\sin(4e^{2x} - 4xe^x + x^2 - 4)$

$\dfrac{dy}{dx} = \cos(4e^{2x} - 4xe^x + x^2 - 4)$

$(8e^{2x} - 4e^x - 4xe^x + 2x - 0)$

$\left.\dfrac{dy}{dx}\right|_{x=0} = \cos(4(1) - 0 + 0 - 4)$

$(8(1) - 4(1) - 0 + 0)$

$= \cos(0)\ (4)$

$= 1(4)$

$\left.\dfrac{dy}{dx}\right|_{x=0} = 4$

Q8

If $g(2x+1) = \sqrt{x^2 + 8x}$, then $g'(3) =$

(a) $\dfrac{5}{3}$

(b) $\dfrac{5}{6}$

(c) $\dfrac{7}{2\sqrt{37}}$

(d) $\dfrac{7}{\sqrt{37}}$

(e) $\dfrac{5}{12}$

$x^2 = (2x+1)^2$

$= 4x^2 + 4x + 1$

$g(2x+1) = \sqrt{4x^2 + 12x + 1}$

$g'(3) = \dfrac{8x + 12}{2\sqrt{4x^2 + 12x + 1}}$

$\dfrac{8(3) + 12}{2\sqrt{36 + 48 + 1}}$

$= \dfrac{36}{2\sqrt{75}}$

$= \dfrac{36}{10\sqrt{3}}$

$\dfrac{36}{2\sqrt{36 + 36 + 1}}$

$\dfrac{18}{\sqrt{73}}$