

Differentiate the following functions

a) $y = \sqrt{\frac{x-2}{x+3}}$, b) $y = \sqrt[3]{(x-2)^2(x+2)}$

c) $y = 6 \ln \frac{x}{\sqrt{2x+1}}$, d) $y = e^{x^2 \ln x^2}$

Solution

a) $y = \left(\frac{x-2}{x+3}\right)^{\frac{1}{2}}$. So $y' = \frac{1}{2} \cdot \frac{d}{dx} \left(\frac{x-2}{x+3}\right) \cdot \left(\frac{x-2}{x+3}\right)^{\frac{1}{2}-1}$. Thus

$$y' = \frac{1}{2} \frac{x+3 - (x-2)}{(x+3)^2} \left(\frac{x-2}{x+3}\right)^{-\frac{1}{2}} = \frac{5}{2(x+3)^2} \left(\frac{x-2}{x+3}\right)^{-\frac{1}{2}}$$

b) $y = \left[(x-2)^2(x+2)\right]^{\frac{1}{3}}$. So $y' = \frac{1}{3} \cdot \frac{d}{dx} [(x-2)^2(x+2)] \cdot \left[(x-2)^2(x+2)\right]^{\frac{1}{3}-1}$

$$= \frac{1}{3} \left[2(x-2)(x+2) + (x-2)^2 \right] \left[(x-2)^2(x+2)\right]^{-\frac{2}{3}}$$

$$= \frac{1}{3} (3x+2) \left[(x-2)^2(x+2)\right]^{-\frac{2}{3}}$$

c) $y = 6 \left[\ln x - \ln \sqrt{2x+1} \right] = 6 \left[\ln x - \frac{1}{2} \ln(2x+1) \right]$. So

$$y' = 6 \left[\frac{1}{x} - \frac{1}{2} \frac{2}{2x+1} \right] = 6 \left[\frac{1}{x} - \frac{1}{2x+1} \right] = 6 \frac{x+1}{x(2x+1)}$$

d) $y = e^{2x^2 \ln x}$. So $y' = \frac{d}{dx} (2x^2 \ln x) \cdot e^{2x^2 \ln x}$

$$= 2 \left[2x \ln x + x^2 \frac{1}{x} \right] e^{2x^2 \ln x} = 2x(2 \ln x + 1) e^{x^2 \ln x^2}$$