

Ex 1: Use the definition of the derivative to find:

a) $f'(x)$ if $f(x) = x^2 + 4x - 8$

b) $g'(x)$ if $g(x) = \frac{6}{x}$

Ex 2 Find an equation of the tangent line to the curve

$$f(x) = -9x^{1/3} + 5x^{-2/5} \text{ at } x_0 = 1$$

Solution

Ex 1: a)
$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + 4x + 4h - 8 - x^2 - 4x + 8}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2xh + h^2 + 4h}{h} = \lim_{h \rightarrow 0} 2x + 4 + h = 2x + 4$$

Hence $f'(x) = 2x + 4$

b)
$$\lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h} = \lim_{h \rightarrow 0} \frac{\frac{6}{x+h} - \frac{6}{x}}{h} = \lim_{h \rightarrow 0} \frac{\frac{6x - 6x - 6h}{x(x+h)}}{h}$$

$$= \lim_{h \rightarrow 0} -\frac{6}{x(x+h)} = -\frac{6}{x^2}. \text{ Hence } g'(x) = -\frac{6}{x^2}$$

Ex 2

$$f'(x) = -9 \cdot \frac{1}{3} \cdot x^{1/3 - 1} + 5 \cdot \left(-\frac{2}{5}\right) x^{-2/5 - 1} = -3x^{-2/3} - 2x^{-7/5}$$

Therefore the slope of the tangent line is $f'(1) = -5$.
An equation of the tangent line is then:

$$y = -5(x-1) - 4 = 1 - 5x$$

$f(1) = -4$