Math 132 Quiz 7 Date 12/08/2010

Ex1: Let $f(x) = \sqrt{x}$. $122 = \underbrace{121}_{x} + \underbrace{1}_{dx}$

$f(x+dx) \approx f(x) + f'(x)\,dx$

$f'(x) = \frac{1}{2\sqrt{x}}$, so $f'(121) = \frac{1}{2\sqrt{121}} = \frac{1}{22}$. Therefore

$\sqrt{122} \approx \sqrt{121} + \frac{1}{22} = 11 + \frac{1}{22} = \frac{243}{22}$

Ex2: a) $\int\left[\frac{x}{x^2+1} + \frac{x^5}{(x^6+1)^2}\right]dx = \underbrace{\int \frac{x}{x^2+1}\,dx}_{I} + \underbrace{\int \frac{x^5}{(x^6+1)^2}\,dx}_{J}$

• $I = \int \frac{x}{x^2+1}\,dx = \frac{1}{2}\int \frac{2x}{x^2+1}\,dx = \frac{1}{2}\ln(x^2+1) + C$

• $J = \int \frac{x^5}{(x^6+1)^2}\,dx = \frac{1}{6}\int 6x^5 (x^6+1)^{-2}\,dx = \frac{1}{6}\cdot\frac{1}{-2+1}(x^6+1)^{-2+1} + C$

$= \frac{-1}{6(x^6+1)} + C$. Therefore

$\int\left[\frac{x}{x^2+1} + \frac{x^5}{(x^6+1)^2}\right]dx = \frac{1}{2}\ln(x^2+1) - \frac{1}{6(x^6+1)} + C$

b) $\int\left(-\frac{\sqrt[3]{x^2}}{5} - \frac{7}{2\sqrt{x}} + 6x\right)dx = \int\left(-\frac{1}{5}x^{\frac{2}{3}} - \frac{7}{2\sqrt{x}} + 6x\right)dx$

$= -\frac{1}{5}\,\frac{1}{\frac{2}{3}+1}\,x^{\frac{2}{3}+1} - 7\sqrt{x} + 3x^2 + C$

$= -\frac{3}{25}x^{\frac{5}{3}} - 7\sqrt{x} + 3x^2 + C$

Date No.

c) $\displaystyle\int \frac{x+1}{x^2+2x} \ln(x^2+2x)\, dx.$

Let $u = \ln(x^2+2x)$. Then $du = \dfrac{2x+2}{x^2+2x}\, dx = 2\,\dfrac{x+1}{x^2+2x}\, dx$

So $\dfrac{x+1}{x^2+2x}\, dx = \dfrac{1}{2}\, du$. Thus

$$\int \frac{x+1}{x^2+2x} \ln(x^2+2x)\, dx = \int \frac{1}{2}\, u \; du = \frac{1}{2}\int u\, du = \frac{1}{2}\,\frac{u^2}{2} + C$$

$$= \frac{1}{4} \ln^2(x^2+2x) + C$$

Math 132 Quiz 8 Date 12/18/2010

Ex 1: Determine the following indefinite integrals:

a) $\int \frac{6x^2\sqrt{\ln(x^3+1)^2}}{x^3+1}\,dx$, b) $\int \sqrt{x}\sqrt{(8x)^{3/2}+3}\,dx$

Ex 2: Find the area of the region bounded by $y = x^3 + x$, $y = 0$ $x = -1$ and $x = 2$.

Solution

Ex 1: a) $\sqrt{\ln(x^3+1)^2} = \sqrt{2\ln(x^3+1)}$. Let $u = 2\ln(x^3+1)$. Then $du = \frac{6x^2}{x^3+1}dx$. Therefore,

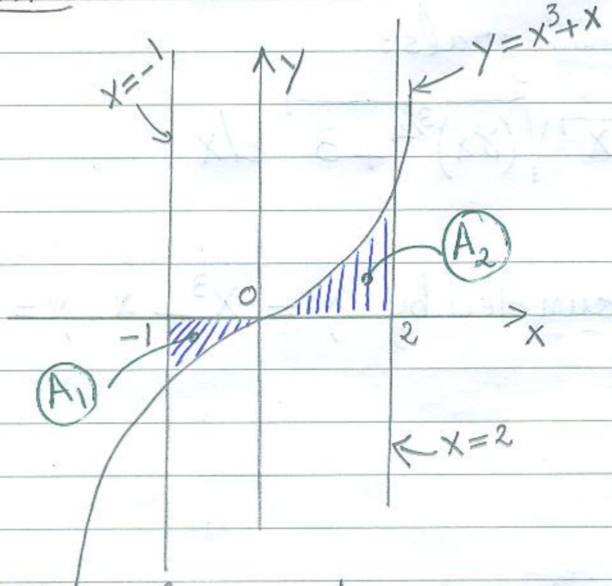
$$\int \frac{6x^2}{x^3+1}\sqrt{\ln(x^3+1)^2}\,dx = \int \sqrt{u}\,du = \frac{2}{3}u^{3/2} + C = \frac{2}{3}\left[2\ln(x^3+1)\right]^{3/2} + C$$

b) Let $u = (8x)^{3/2} + 3$, then $du = \frac{3}{2}\cdot 8\cdot(8x)^{\frac{1}{2}}dx$, so $du = 12\sqrt{8}\sqrt{x}\,dx$. Thus $\sqrt{x}\,dx = \frac{1}{24\sqrt{2}}du$. Therefore

$$\int \sqrt{x}\sqrt{(8x)^{3/2}+3}\,dx = \frac{1}{24\sqrt{2}}\int \sqrt{u}\,du = \frac{1}{24\sqrt{2}}\,\frac{2}{3}\,u^{3/2} + C$$

$$= \frac{1}{36\sqrt{2}}\left[(8x)^{3/2}+3\right]^{3/2} + C$$

Ex2 :



$\circ)\ A_1 = \int_{-1}^{0} -(x^3+x)\,dx = \int_{0}^{-1}(x^3+x)\,dx$

$= \left.\dfrac{x^4}{4} + \dfrac{x^2}{2}\right|_0^{-1} = \dfrac{1}{4} + \dfrac{1}{2}$

$= \dfrac{3}{4}$

$\circ)\ A_2 = \int_0^2 (x^3+x)\,dx$

$= \left.\dfrac{x^4}{4} + \dfrac{x^2}{2}\right|_0^2$

$= 4 + 2 = 6$

Therefore $A = A_1 + A_2 = 6 + \dfrac{3}{4} = \dfrac{27}{4}$.

Math 132 Quiz 9 Date 12/25/2010

Ex 1. a) $I = \int 3(2x-2)\ln(x-2)\,dx = 6\int (x-1)\ln(x-2)\,dx$

$$= 6\underbrace{\int x\ln(x-2)\,dx}_{K} - 6\underbrace{\int \ln(x-2)\,dx}_{L}$$

*) $K = \int x\ln(x-2)\,dx$. Let $u' = x \Rightarrow u = \frac{1}{2}x^2$

$v = \ln(x-2) \Rightarrow v' = \frac{1}{x-2}$

$$K = \frac{1}{2}x^2\ln(x-2) - \frac{1}{2}\int \frac{x^2}{x-2}\,dx$$

$$= \frac{x^2}{2}\ln(x-2) - \frac{1}{2}\int\left(x+2+\frac{4}{x-2}\right)dx$$

$$= \frac{x^2}{2}\ln(x-2) - \frac{1}{2}\left(\frac{x^2}{2} + 2x + 4\ln(x-2)\right) + C$$

$$= \left(\frac{x^2}{2} - 2\right)\ln(x-2) - \frac{x^2}{4} - x + C$$

*) $L = \int \ln(x-2)\,dx$. Let $u' = 1 \Rightarrow u = x$

$v = \ln(x-2) \Rightarrow v' = \frac{1}{x-2}$.

$$L = x\ln(x-2) - \int \frac{x}{x-2}\,dx = x\ln(x-2) - \int \frac{x-2+2}{x-2}\,dx$$

$$= x\ln(x-2) - \int\left(1 + \frac{2}{x-2}\right)dx = x\ln(x-2) - x - 2\ln(x-2) + C$$

$$= (x-2)\ln(x-2) - x + C.$$

Therefore:

$$I = 6\left(\frac{x^2}{2} - 2\right)\ln(x-2) - \frac{3}{2}x^2 - 6x - 6(x-2)\ln(x-2) + 6x + C$$

$$= 6\left(\frac{x^2}{2} - x\right)\ln(x-2) - \frac{3}{2}x^2 + C = 6x\left(\frac{x}{2} - 1\right)\ln(x-2) - \frac{3}{2}x^2 + C$$

b) $J = \int x^3 e^{x^2} dx = \int x^2 \, x e^{x^2} dx$

Let $u = x^2 \Rightarrow u' = 2x$

$v' = x e^{x^2} \Rightarrow v = \frac{1}{2} e^{x^2}$

$$J = \frac{x^2}{2} e^{x^2} - \int x e^{x^2} dx = \frac{x^2}{2} e^{x^2} - \frac{1}{2} e^{x^2} + c$$

$$= \frac{1}{2}(x^2 - 1) e^{x^2} + C$$

Ex 2: a) $\int \tan^3 x \, dx = \int \tan x . \tan^2 x \, dx = \int \tan x \, (1 + \tan^2 x - 1) dx$

$$= \int \tan x . (1 + \tan^2 x) dx - \int \tan x \, dx$$

$$= \frac{1}{2} \tan^2 x + \ln|\cos x| + C$$

b) $\int \sin^2 x \cos^2 x \, dx = \int \frac{1 - \cos 2x}{2} \, \frac{1 + \cos 2x}{2} \, dx$

$$= \frac{1}{4} \int (1 - \cos 2x)(1 + \cos 2x) \, dx$$

$$= \frac{1}{4} \int (1 - \cos^2 2x) dx = \frac{1}{4} \int \left(1 - \frac{1 + \cos 4x}{2}\right) dx$$

$$= \frac{1}{4} \int \left(\frac{1}{2} - \frac{1}{2} \cos 4x\right) dx = \frac{1}{4} \left[\frac{x}{2} - \frac{\sin 4x}{8}\right] + C$$

$$= \frac{1}{8} \left(x - \frac{\sin 4x}{4}\right) + C$$

Math 132 Quiz 10 Date 01/01/2011

Find the indicated partial derivatives.

a) $f(x,y) = (x+y)^2(xy)$.

$f_x(x,y)$, $f_y(x,y)$, $f_{xx}(x,y)$, $f_{yy}(x,y)$ and $f_{xy}(x,y)$.

b) $z = e^{\sqrt{x^2+y^2}}$. $\frac{\partial z}{\partial y}$, $\frac{\partial^2 z}{\partial y^2}$ and $\frac{\partial^2 z}{\partial x \partial y}$

Solution

a) *) $f_x(x,y) = 2(x+y)(xy) + y(x+y)^2 = y(x+y)\left[2x + x+y\right]$

$= y(x+y)(3x+y)$

*) $f_y(x,y) = x(x+y)(3y+x)$. In fact x and y play symmetric role, so to find $f_y(x,y)$ just replace in the expression of $f_x(x,y)$ x by y and y by x (in other words permute the role of x and y in $f_x(x,y)$)

*) $f_{xx}(x,y) = y(3x+y) + 3y(x+y) = y(6x+4y)$

*) $f_{yy}(x,y) = x(6y+4x)$

*) $f_{xy}(x,y) = (x+y+y)(3x+y) + y(x+y) = (x+2y)(3x+y) + y(x+y)$

b)

*) $\frac{\partial z}{\partial y} = \frac{1}{2}\, 2y\,(x^2+y^2)^{\frac{1}{2}-1}\, e^{\sqrt{x^2+y^2}} = \frac{y}{\sqrt{x^2+y^2}}\, e^{\sqrt{x^2+y^2}}$

*) $\frac{\partial^2 z}{\partial y^2} = \left[\frac{1}{\sqrt{x^2+y^2}} - \frac{y^2}{(x^2+y^2)^{3/2}}\right] e^{\sqrt{x^2+y^2}} + \frac{y^2}{x^2+y^2}\, e^{\sqrt{x^2+y^2}}$

$= \left[\frac{x^2}{(x^2+y^2)^{3/2}} + \frac{y^2}{x^2+y^2}\right] e^{\sqrt{x^2+y^2}} = \left(\frac{x^2}{\sqrt{x^2+y^2}} + y^2\right) \frac{e^{\sqrt{x^2+y^2}}}{x^2+y^2}$

Date No.

*) $$\frac{\partial^2 z}{\partial x \partial y} = y\left[-\frac{1}{2}2x\left(x^2+y^2\right)^{-\frac{1}{2}-1} e^{\sqrt{x^2+y^2}} + \frac{1}{\sqrt{x^2+y^2}}\,\frac{2x}{2\sqrt{x^2+y^2}}\,e^{\sqrt{x^2+y^2}}\right]$$

$$= xy\left[\frac{1}{x^2+y^2} - \frac{1}{\left(x^2+y^2\right)^{3/2}}\right] e^{\sqrt{x^2+y^2}}$$

$$= \frac{xy}{x^2+y^2}\left[1 - \frac{1}{\sqrt{x^2+y^2}}\right] e^{\sqrt{x^2+y^2}}$$