MATH 201 2010/11. Calculus III.
Quiz 2 Questions §10.1 -- §12.4

Important note: show your workings out for full marks.

1. Show that the equation represents a sphere and find its center and radius:
   \[ x^2 + y^2 + z^2 = 4x - 2y + 6z \]

2. Determine whether the following pairs of vectors are orthogonal, parallel or neither:
   (a) \( \mathbf{a} = (-5,3,7) \) and \( \mathbf{b} = (6,-8,2) \);
   (b) \( \mathbf{a} = (4,5,-1) \) and \( \mathbf{b} = (-3,2,-2) \); and
   (c) \( \mathbf{a} = 2i - 6j + 4k \) and \( \mathbf{b} = 3i - 9j + 6k \).

3. Find the volume of the parallelepiped with adjacent edges, \( \mathbf{AB} \), \( \mathbf{AC} \), and \( \mathbf{AD} \) where \( A(2,01), B(4,1,0), C(3,-1,1), D(2,-2,2) \).

4. Find \( \frac{dy}{dx} \) and \( \frac{d^2y}{dx^2} \) at the point \( s=1/2 \) for the curve given by \( x = 2s^3, \quad y = 6/s \).

5. Find the area of the region between the two polar curves \( r = 2 \sin 2\theta \), and \( r = 1 \), in the range \( 0 \leq \theta \leq \pi/2 \).

6. If \( \mathbf{a} = (1,2,3) \) and \( \mathbf{b} = (1,0,1) \), show that, (a) \( \mathbf{u} = \mathbf{b} - \text{Proj}_a \mathbf{b} \) is orthogonal to \( \mathbf{a} \); (b) \( \mathbf{a} \) is parallel to \( \text{Proj}_a \mathbf{b} \).

7. Find the Cartesian (rectangular) equations of the curve given by the parametric equations \( x(t) = \cos t, \quad y(t) = \cos 2t - 1; \quad -\pi \leq t \leq \pi \).