MATH 202.6 (101)

Quiz 3 (Chaps. 4.1.3-4.5) Duration: 20mn

Name: ID number:

1.)(3pts)Knowing that $y_1 = x$ is a solution to the DE

$$x^2y'' - xy' + y = 0,$$

find a second solution y_2 linearly independent to y_1 by using only the method of reduction of order.

2.)(7pts)Solve the following non homogeneous linear DE with constant coefficients

$$y''' - 2y'' - y' + 2y = 1.$$

Solution

1) $y_2 = y_1 \int \frac{e^{-\int P(x)dx}}{y_1^2} dx$

$y'' - \frac{1}{x} y' + \frac{1}{x^2} y = 0$

$P(x) = -\frac{1}{x} \Rightarrow e^{-\int P(x)dx} = e^{\int \frac{1}{x}dx} = e^{\ln x} = x,\ x>0$

$y_2 = x \int \frac{x}{x^2} dx = x \int \frac{dx}{x} = x \ln x$

$y_2 = x \ln x,\ x > 0.$

2) $y = y_c + y_p$, where y_c is the general solution of the associated homogeneous equation

$y''' - 2y'' - y' + 2y = 0$:

The auxiliary equation associated with this equation is

$m^3 - 2m^2 - m + 2 = 0$

$(m-1)(m^2 - m - 2) = 0$

$m = 1,\ m = -1,\ m = 2$

$y_c = c_1 e^{x} + c_2 e^{-x} + c_3 e^{2x}.$

Now, it is easy to see that $y_p = \frac{1}{2}$ is a particular solution.

Therefore,

$$\boxed{y = c_1 e^{x} + c_2 e^{-x} + c_3 e^{2x} + \frac{1}{2}}$$