Name: ___________________________ ID#: _______________
Instructor: ______________________ Sec #: _______ Serial #: _______

- Mobiles and calculators are not allowed in this exam.
- Write all steps clear.

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<th>Question #</th>
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Q: 1 (a) (5 points) Find Laplace transform $\mathcal{L}\{f(t)\}$ where

$$f(t) = \begin{cases} 0 & \text{if } 0 \leq t < 1 \\ t^2 & \text{if } t \geq 1 \end{cases}.$$ 

(b) (5 points) Find inverse Laplace transform $\mathcal{L}^{-1}\left\{ \frac{e^{-2s}}{s^2(s-1)} \right\}$. 
Q:2 (12 points) Let $\vec{F}(x, y, z) = \frac{x}{x^2 + y^2 + z^2} \vec{i} + \frac{y}{x^2 + y^2 + z^2} \vec{j} + \frac{z}{x^2 + y^2 + z^2} \vec{k}$ and $D$ is the region bounded by the concentric spheres $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 + z^2 = 4$. Use divergence theorem to evaluate $\iint_S (\vec{F} \cdot \hat{n}) dS$. 
Q:3 (16 points) Use separation of variables method to find the nontrivial solution of the heat equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad 0 < x < 1, \quad t > 0$$

subject to the boundary and initial conditions

$$u(0, t) = 0, \quad u(1, t) = 0, \quad t > 0$$
$$u(x, 0) = 10, \quad 0 < x < 1.$$
Q:4 (20 points) Use separation of variables method to find the nontrivial solution of the wave equation

\[ \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} = \frac{\partial^2 u}{\partial t^2}, \quad 0 < r < 2, \quad t > 0 \]

subject to the boundary conditions

\[ u(2, t) = 0, \quad t > 0 \]
\[ u(r, 0) = 1, \quad \frac{\partial u}{\partial t} \bigg|_{t=0} = 0, \quad 0 < r < 2 \]

solution is bounded at \( r = 0 \).
Q:5 (20 points) Find the steady-state temperature $u(r, \theta)$ in a sphere of radius 2 by solving the problem

$$\frac{\partial^2 u}{\partial r^2} + \frac{2}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} + \frac{\cot \theta}{r^2} \frac{\partial u}{\partial \theta} = 0, \quad 0 < r < 2, \ 0 < \theta < \pi,$$

subject to the boundary condition

$$u(2, \theta) = 1 + \cos(\theta), \quad 0 < \theta < \pi.$$
Q:6 (15 points) Use Laplace transform to solve the problem

\[ \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \quad 0 < x < L, \ t > 0 \]

subject to the boundary and initial conditions

\[ u(0, t) = 0, \quad u(L, t) = 0, \quad t > 0 \]

\[ u(x, 0) = 2 \sin \left( \frac{\pi x}{L} \right), \quad \frac{\partial u}{\partial t} \bigg|_{t=0} = 0, \quad 0 < x < L. \]
Q:7 (15 points) Use appropriate Fourier transform to solve the problem

\[
\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad x > 0, \ t > 0
\]

subject to the conditions

\[
\begin{align*}
u(0, t) &= 5, \ t > 0 \\
u(x, 0) &= 0, \ x > 0.
\end{align*}
\]