King Fahd University of Petroleum and Minerals  
Department of Mathematics and Statistics

Math 302 Exam II  
Semester (101) December 9, 2010 Time: 12:30 - 14:15 pm

Name: ........................................ I.D: ................. Section: ...

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Problem 1.

(a) Let $\varphi$ be the scalar field defined by $\varphi(x, y, z) = x^2yz$. Compute $\text{grad}(\varphi)$ and $\text{curl}(\text{grad}(\varphi))$.

(b) Let $F$ be the vector field defined by

$$F(x, y, z) = (y + e^y, z + e^z, x + e^x).$$

Is there a scalar field $\psi$ with continuous first and second partial derivatives such that $F = \text{grad}(\psi)$?
Problem 2. Find the length of the curve $C$ given by

$$y = 2x^2 - \frac{1}{16} \ln x, \quad 1 \leq x \leq 3.$$
Problem 3. Let $C = C_1 \cup C_2 \cup C_3$ be the positively oriented closed path in $\mathbb{R}^2$, with 3 smooth pieces:

- $C_1$ : the straight line segment joining $(0, 0)$ and $(1, 0)$.
- $C_2$ : the quarter circle (of center the origin) joining $(1, 0)$ and $(0, 1)$.
- $C_3$ : the straight line segment joining $(0, 1)$ and $(0, 0)$.

A vector field is given by $F(x, y) = (x + y)i + yj$.

Verify Green’s theorem, by computing the line and the double integrals.
Problem 4. Let $\Sigma$ be the surface of $\mathbb{R}^3$ given by $z = x + 2y^2$, with $0 \leq x \leq 1$ and $0 \leq y \leq \sqrt{6}$. Evaluate the surface integral

$$I = \int \int_{\Sigma} y \, d\sigma.$$
Problem 5. Let $\Sigma$ be the closed surface which consists of the part of the cylinder $z^2 + x^2 = 4$ lying in the first octant and the parts of the planes $y = 0$, $y = 3$, $z = 0$, $x = 0$, as shown in the figure.

Given a field $F$ by $F(x, y, z) = xz^2\mathbf{i} + (x^2 - z + y)\mathbf{j} + zx^2\mathbf{k}$. Evaluate the flux of $F$ across $\Sigma$

$$Q = \iint_{\Sigma} F \cdot n \, d\sigma.$$