KING FAHD UNIVERSITY OF PETROLEUM & MINERALS  
Department of Mathematics & Statistics  

Math 605                   Asymptotic Expansions and Perturbation Methods  

Mid Term Exam                                                       Term 101  

Time Allowed 2  Hours  

Name __________________     ID #  _______  

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Important Note  

Write clearly and show all work.  

Instructor: F. D. Zaman  
#               #
Q1) Find asymptotic solution of the differential equation

\[ u'' + 2u' + \frac{2}{x}u = 0, \ x \to \infty \]

In the form

\[ e^{2x} x^{-\sigma} g(x) \]

Where \( g(x) \) is a suitable asymptotic function to be determined.
Q2) Seek asymptotic solution of

\[ u'' - x(x + 2)u = 0, \quad \text{as} \quad x \to \infty \]

In the form

\[ e^{\phi_0(x)} + e^{\phi_1(x)} + \cdots \]

\[ x \to \infty \]

where \( \{\phi_n(x)\} \) is an asymptotic sequence as \( x \to \infty \).
Q 3) Solve the boundary value problem
\[ \frac{d^2 u}{dx^2} + (\lambda + x)u = 0, \quad 0 < x < \infty \]
\[ u(0, \lambda) = a, \quad u'(0, \lambda) = b \]
Where a,b are constants. \( \lambda \) and \( x \) are large.