KFUPM SEM II (Term 102) Name: Solutions Serial #: ________

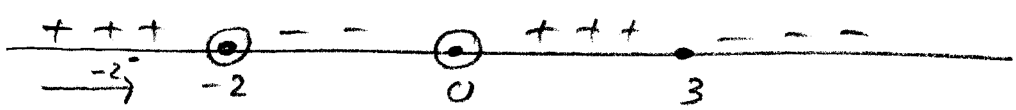
MATH 101-2-4 Quiz # 1 ID: #: ________________ Sec. #: ________

5 points each

1. Evaluate $\lim\limits_{x\to -2^-} \dfrac{3-x}{x^2+2x}$. [Your answer must be a number, or ∞, or $-\infty$. Use a sign diagram to justify your answer].

It is a problem of $\frac{5}{0}$, so we have to decide whether the limit is $+\infty$ or $-\infty$.

The sign diagram of $\dfrac{3-x}{x(x+2)}$ is



$$\Rightarrow \lim_{x\to -2^-} \frac{3-x}{x^2+2x} = \infty$$

2. If $\lim\limits_{x\to 2} f(x) = L$ and $9 - 5x^2 \le -3x + 5f(x) \le \dfrac{x^2-13x}{4x-6}$, for all x near 2, find the value of L.

We have $\lim\limits_{x\to 2}(9-5x^2) = 9 - 20 = -11$

and $\lim\limits_{x\to 2} \dfrac{x^2-13x}{4x-6} = \dfrac{-22}{2} = -11$

Therefore $\lim\limits_{x\to 2}(-3x + 5f(x)) = -11$ by the Squeezing Theorem $\Rightarrow \lim\limits_{x\to 2}(-3x) + \lim\limits_{x\to 2} 5f(x) = -11$

$\Rightarrow -6 + 5L = -11 \Rightarrow 5L = -5 \Rightarrow L = -1$

3. Evaluate $\lim_{x \to -3} \frac{x+3}{2-\sqrt{2x+10}}$. $\left(\frac{0}{0} \text{ type}\right)$

$$\lim_{x \to -3} \frac{x+3}{2-\sqrt{2x+10}} = \lim_{x \to -3} \frac{(x+3)\left[2+\sqrt{2x+10}\right]}{4-(2x+10)}$$

$$= \lim_{x \to -3} \frac{(x+3)\left(2+\sqrt{2x+10}\right)}{-2x-6}$$

$$= \lim_{x \to -3} \frac{\cancel{(x+3)}\cdot\left(2+\sqrt{2x+10}\right)}{-2\cancel{(x+3)}}$$

$$= \lim_{x \to -3} \frac{2+\sqrt{2x+10}}{-2} = \frac{2+2}{-2} = -2$$

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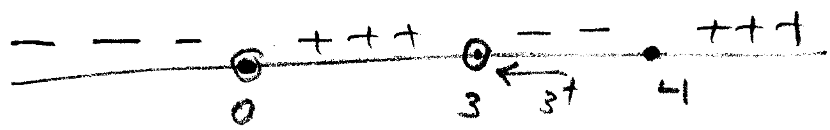
1. Evaluate $\lim_{x \to 3^+} \frac{x-4}{x^2-3x}$. [Your answer must be a number, or ∞, or $-\infty$. Use a sign diagram to justify your answer].

It in a problem of $\frac{-1}{0}$, so we have to decide whether the limit in $+\infty$ or $-\infty$.

The sign diagram of $\frac{x-4}{x(x-3)}$ is:



$$\Rightarrow \lim_{x \to 3^+} \frac{x-4}{x^2-3x} = -\infty$$

2. If $\lim_{x \to 2} f(x) = L$ and $-2x^2+7 \le 8x+4f(x) \le \frac{x^2+3x}{x-12}$, for all x near 2, find the value of L.

We have $\lim_{x \to 2} (-2x^2+7) = -1$

and $\lim_{x \to 2} \left(\frac{x^2+3x}{x-12}\right) = \frac{10}{-10} = -1$

Therefore $\lim_{x \to 2} (8x+4f(x)) = -1$ by the squeezing Theorem $\Rightarrow$

$\lim_{x \to 2} (8x) + \lim_{x \to 2} 4f(x) = 16 + 4L = -1 \Rightarrow$

$4L = -17 \Rightarrow L = \frac{-17}{4}$.

• 3. Evaluate $\lim_{x \to 2} \frac{2-x}{\sqrt{3x-2}-2}$. ($0/0$ type)

$$\lim_{x \to 2} \frac{2-x}{\sqrt{3x-2}-2} = \lim_{x \to 2} \frac{(2-x)(\sqrt{3x-2}+2)}{(3x-2)-4}$$

$$= \lim_{x \to 2} \frac{(2-x)(\sqrt{3x-2}+2)}{3x-6}$$

$$= \lim_{x \to 2} \frac{\overset{-1}{\cancel{(2-x)}}(\sqrt{3x-2}+2)}{3\cancel{(x-2)}}$$

$$= \lim_{x \to 2} -\frac{1}{3}(\sqrt{3x-2}+2)$$

$$= -\frac{1}{3}(\sqrt{4}+2) = -\frac{4}{3}.$$