King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics

CODE 004
Math 101
Exam 2
Term 102
Monday, April 25, 2011
Net Time Allowed: 120 minutes

Name: ____________________________
ID: _______________ Sec: ____________

Check that this exam has 20 questions.

Important Instructions:

1. All types of calculators, pagers or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.
1. If \( x^3 + x^2y - 4y^2 = 6 \), then \( y' = \)

(a) \( \frac{-3x^2}{x^2 - 8y} \)

(b) \( \frac{x + 3xy}{4y^2 + 4x} \)

(c) \( \frac{xy - 3x^2}{4y - 2x} \)

(d) \( \frac{2xy + 3x^2}{8y - x^2} \)

(e) \( \frac{3x^2 + xy}{8y} \)

2. The \( x \)-coordinates of the points on the curve \( y = 3x - \sin x \) at which the tangent line has slope 4 are

(a) \( n\pi, \ n \) is integer

(b) \( 2n\pi, \ n \) is integer

(c) \( \frac{2n + 1}{2}\pi, \ n \) is integer

(d) \( \frac{n\pi}{3}, \ n \) is integer

(e) \( (2n + 1)\pi, \ n \) is integer
3. The equation of the normal line to the curve 
\( y = 4x^3 - 6\sqrt{x} \) at the point \((1, -2)\) is

(a) \( y + 9x - 17 = 0 \)

(b) \( 9y + x + 17 = 0 \)

(c) \( y = 17x - 9 \)

(d) \( y + 9x - 11 = 0 \)

(e) \( y = 9x + 11 \)

4. The slope of the tangent line to the curve \( y = 4\sec x + \tan x \)
at \( x = \frac{\pi}{4} \) is equal to

(a) \( 1 + \sqrt{2} \)

(b) \( 2 \)

(c) \( -4\sqrt{2} \)

(d) \( -6 \)

(e) \( 2 + 4\sqrt{2} \)
5. If \( f(x) = \sqrt[3]{x^2} + \frac{1}{\sqrt[x]{x}} \), then \( f''(1) = \)

\(\text{(a) } \frac{13}{144} \)

\(\text{(b) } \frac{-3}{56} \)

\(\text{(c) } \frac{13}{56} \)

\(\text{(d) } \frac{101}{144} \)

\(\text{(e) } \frac{77}{144} \)

6. \( \lim_{x \to 0} \frac{\cos x + \sin(2x) - 1}{\tan x} = \)

\(\text{(a) } 2 \)

\(\text{(b) } \text{Does not exist} \)

\(\text{(c) } -3 \)

\(\text{(d) } 1 \)

\(\text{(e) } 0 \)
7. If \( f(x) = \frac{2x - 1}{(x + 3)^3} \), then \( f'(x) = \)

(a) \( \frac{3 + 2x}{(x + 3)^4} \)

(b) \( \frac{5 + 4x}{(x + 3)^5} \)

(c) \( \frac{9 - 4x}{(x + 3)^4} \)

(d) \( \frac{3x + 2}{(x + 3)^6} \)

(e) \( \frac{7 - 4x}{(x + 3)^5} \)

8. If \( y = \sqrt[3]{u^2 - 3} \) and \( u = \sqrt{x^2 - 1} \), then \( \frac{dy}{dx} \bigg|_{x=3} = \)

(a) \( \frac{8}{\sqrt[3]{2}} \)

(b) \( \frac{1}{5} \)

(c) \( \frac{2}{5} \)

(d) \( \frac{8}{15} \)

(e) \( \frac{3}{\sqrt[4]{4}} \)
9. If \( h(x) = \ln(x + 2 \ln x) \), then \( h'(x) = \)

(a) \( \frac{x + 1}{x^2 + 2 \ln x} \)

(b) \( \frac{1}{x + 2 \ln x} \)

(c) \( \frac{2}{x(x + 2 \ln x)} \)

(d) \( \frac{x + 2}{x^2 + 2x \ln x} \)

(e) \( \frac{x + 2}{x + 2 \ln x} \)

10. If \( f(t) = 3^{\sin^2(3t)} \), then \( f'(t) = \)

(a) \( \sin^2(3t) \cdot 3^{\sin^2(3t)-1} \)

(b) \( 3^{\sin^2(3t)} \cdot \ln 3 \cdot 3 \sin(3t) \cdot \cos(3t) \)

(c) \( 3^{\sin^2(3t)} \cdot \ln 9 \cdot 3 \cos(3t) \)

(d) \( 3^{\sin^2(3t)} \cdot \ln 3 \cdot 2 \sin(3t) \)

(e) \( 3^{\sin^2(3t)} \cdot \ln(27) \cdot \sin(6t) \)
11. Let \( f \) be a differentiable function such that \( f(2) = 2, f(4) = 1, f'(2) = 3, \) and \( f'(4) = -1. \) If \( G(x) = f(2x) \cdot f(x), \) then \( G'(2) = \)

(a) \(-1\)
(b) 0
(c) 5
(d) 3
(e) 1

12. The equation of the tangent line to the curve \( x^2 + (y - x)^3 = 9 \) at \( x = 1 \) is

(a) \( y = \frac{5}{6}x + \frac{13}{6} \)
(b) \( y = \frac{5}{3}x + \frac{4}{3} \)
(c) \( y = \frac{1}{6}x + \frac{17}{6} \)
(d) \( y = \frac{7}{6}x + \frac{11}{6} \)
(e) \( y = \frac{3}{6}x - \frac{17}{6} \)
13. If \( y = x\sin^{-1}x \), then \( \frac{y'}{y} = \)

(a) \( \frac{\sin^{-1}x}{x} + \frac{\ln x}{\sqrt{1-x^2}} \)

(b) \( x^{\sin^{-1}x} \cdot \ln x \)

(c) \( \frac{1}{x\sqrt{1-x^2}} \)

(d) \( (\sin^{-1}x) \ln x \)

(e) \( \frac{\sin^{-1}x}{\sqrt{1-x^2}} + \frac{\ln x}{x} \)

14. \( \lim_{t \to 0} \frac{\sin^2(3t)}{t^3 - 3t^2} = \)

(a) \( \frac{2}{3} \)

(b) \( \frac{1}{9} \)

(c) \( 3 \)

(d) \( \frac{4}{27} \)

(e) \( -3 \)
15. If \( y = mx + k \) is the equation of a line parallel to the line \( y = x \) and tangent to the graph of \( y = e^{x+2} \), then \( m + k = \)

(a) \(-2\)
(b) 5
(c) 3
(d) 2
(e) 4

16. The volume of a cube is increasing at a rate of 10 cm\(^3\)/min. When the length of an edge is 30 cm, the surface area of the cube is increasing at a rate of

(a) \(5 \text{ cm}^2/\text{min}\)
(b) \(6 \text{ cm}^2/\text{min}\)
(c) \(\frac{5}{3} \text{ cm}^2/\text{min}\)
(d) \(\frac{4}{3} \text{ cm}^2/\text{min}\)
(e) \(\frac{5}{9} \text{ cm}^2/\text{min}\)
17. If \( y = \tan^{-1}\left(\frac{1+x}{1-x}\right) \), then \( \frac{dy}{dx} = \)

(a) \( \frac{2}{2+x^2} \)

(b) \( \frac{1}{1+x^2} \)

(c) \( \frac{2}{(1+x)^2} \)

(d) \( \frac{(1-x)^2}{1-(1+x)^2} \)

(e) \( \frac{(1+x)^2}{(1-x)^2} \)

18. A parabola \( y = ax^2 + bx + c \) passes through the point \((1, 7)\), has a tangent line at \( x = -1 \) with slope 6, and has a tangent line at \( x = 5 \) with slope -2. The value of \( 6a + 3b + c \) is equal to

(a) 13

(b) 7

(c) -8

(d) 0

(e) -12
19. If \( f(x) = \frac{\sqrt{3x - 2}}{e^{x^2}(x^3 + 1)^{10}} \), then \( f'(1) = \)

[Hint: you may use logarithmic differentiation]

(a) \( \frac{-16}{5e} \)

(b) \( \frac{-1}{64e} \)

(c) \( \frac{e}{15} \)

(d) \( \frac{32}{e} \)

(e) \( \frac{1}{16e} \)

20. A particle moves according to the law of motion

\[ f(t) = 9te^{-t^3/3}, \quad 0 \leq t \leq 8. \]

The time interval(s) on which the particle is slowing down is(are)

(a) (0, 2) and (3, 6)

(b) (0, 3) and (4, 8)

(c) (0, 3) and (6, 8)

(d) (2, 6)

(e) (3, 6)
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