

King Fahd University of Petroleum and Minerals

Dept. of Mathematics

MATH 102

Quiz 3

Name: _____ ID# _____

1. Evaluate the integral

$$\int_e^{e^2} \frac{(\ln x)^2}{x} dx$$

Let $u = \ln x \Rightarrow du = \frac{1}{x} dx \Rightarrow \boxed{x du = dx}$

when $x = e \Rightarrow \boxed{u = \ln e = 1}$, when $x = e^2 \Rightarrow u = \ln e^2 = 2 \ln e = 2$

$$\begin{aligned} \Rightarrow \int_e^{e^2} \frac{(\ln x)^2}{x} dx &= \int_1^2 \frac{u^2}{x} \cdot x du = \int_1^2 u^2 du = \left. \frac{u^3}{3} \right|_1^2 = \frac{2^3}{3} - \frac{1^3}{3} \\ &= \frac{8}{3} - \frac{1}{3} = \boxed{\frac{7}{3}} \end{aligned}$$

$$\int \frac{\sin(\tan x)}{\cos^2 x}$$

Let $u = \tan x \Rightarrow du = \sec^2 x dx = \frac{1}{\cos^2 x} dx$,

$\Rightarrow \boxed{\cos^2 x du = dx}$

$$\Rightarrow \int \frac{\sin(\tan x)}{\cos^2 x} dx = \int \frac{\sin(u)}{\cos^2 x} \cdot \cos^2 x du = \int \sin u du$$

$$\int \frac{5x}{1+16x^4} dx$$

Solution:

$$\int \frac{5x}{1+16x^4} dx = \int \frac{5x}{1+(4x^2)^2} dx$$

$$\text{Let } u = 4x^2 \Rightarrow du = 8x dx \Rightarrow$$

$$\boxed{\frac{du}{8x} = dx}$$

$$= \int \frac{5x}{1+u^2} \frac{du}{8x} = \frac{5}{8} \int \frac{du}{1+u^2}$$

$$= \frac{5}{8} \tan^{-1}(u) + C$$

$$\boxed{= \frac{5}{8} \tan^{-1}(4x^2) + C}$$