Math 102
Exam I
102
Saturday, March 26, 2011

This exam was prepared using mcqs
For questions send an email to Dr. Ibrahim Al-Lehyani (iallehyani@kaau.edu.sa)
Math 102
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102
Saturday, March 26, 2011
Net Time Allowed: 120 minutes

MASTER VERSION
1. Using four rectangles and midpoint approximation, the area under the graph of \( y = x^2 \) from 1 to 9 is approximately equal to

(a) 240
(b) 180
(c) 120
(d) 84
(e) 164

2. An expression as a limit for the area under the graph of the function \( y = x \cos x \) on \([0, \frac{\pi}{2}]\) is

(a) \( \lim_{n \to +\infty} \sum_{i=1}^{n} \frac{\pi^2 i}{(2n)^2} \cos \left( \frac{\pi i}{2n} \right) \)

(b) \( \lim_{n \to +\infty} \sum_{i=1}^{n} \frac{\pi^2 i}{2n^2} \cos \left( \frac{\pi i}{2n} \right) \)

(c) \( \lim_{n \to +\infty} \sum_{i=1}^{n} \frac{\pi i}{2n^2} \cos \left( \frac{\pi i}{2n} \right) \)

(d) \( \lim_{n \to +\infty} \sum_{i=1}^{n} \frac{\pi^2 i}{n^2} \cos \left( \frac{\pi i}{n} \right) \)

(e) \( \lim_{n \to +\infty} \sum_{i=1}^{1=n} \frac{\pi^2 i}{n} \cos \left( \frac{\pi i}{n} \right) \)
3. The limit \( \lim_{n \to +\infty} \sum_{i=1}^{n} \frac{4}{n} \cosh \left( \frac{4i}{n} + 2 \right) \) can be interpreted as the area under the graph of the function

(a) \( y = \cosh(x + 2), \quad 0 \leq x \leq 4 \)

(b) \( y = 2 \cosh x, \quad 0 \leq x \leq 4 \)

(c) \( y = \cosh \frac{x}{2}, \quad 0 \leq x \leq 4 \)

(d) \( y = \cosh(x + 2), \quad 2 \leq x \leq 4 \)

(e) \( y = \cosh x, \quad 0 \leq x \leq 4 \)

4. If \( f \) is an even function and \( \int_{-2}^{2} f(x) \, dx = 4 \) and \( \int_{0}^{7} f(x) \, dx = 3 \), then \( \int_{-2}^{7} f(x) \, dx \) is

(a) 5

(b) -1

(c) 1

(d) 7

(e) 10
5. If \( f(x) = \begin{cases} -1, & 2 \leq x \leq 3 \\ x - 4, & 3 \leq x \leq 9 \\ 5, & 9 \leq x \leq 10 \end{cases} \), then \( \int_2^{10} f(x) \, dx \) is

(a) 16
(b) 15
(c) 17
(d) 18
(e) 19

6. The value of the integral \( \int_{-4}^{0} (2 + \sqrt{16 - x^2}) \, dx \) is

(a) \( 8 + 4\pi \)
(b) \( 4\pi \)
(c) \( 8 \)
(d) \( 2 + 8\pi \)
(e) \( -8 + 8\pi \)
7. Let \( f(x) = \int_{x^2}^{x^3} \tan^{-1} t \, dt \). Then \( f(1) + f'(1) \) is

(a) \( \frac{\pi}{4} \)
(b) 0
(c) \( \frac{\pi}{2} \)
(d) 2
(e) \( \frac{\pi}{6} \)

8. Let \( F(x) = \int_1^x f(t) \, dt \) where \( f(t) = \int_1^t \frac{\sqrt{1 + u^4}}{u} \, du \). Then \( F''(2) \) is

(a) \( \sqrt{257} \)
(b) 16
(c) \( \sqrt{255} \)
(d) 15
(e) \( \sqrt{270} \)
9. If the velocity of a particle moving along a straight line is given by \( v(t) = \frac{1}{2} - \cos t \), the distance traveled during the interval time \([0, \frac{\pi}{2}]\) is

(a) \( \sqrt{3} - 1 - \frac{\pi}{12} \)
(b) \( \frac{\pi}{4} - 1 \)
(c) \( \sqrt{3} - 1 + \frac{\pi}{12} \)
(d) \( 1 - \frac{\pi}{4} \)
(e) \( \sqrt{3} + 1 - \frac{\pi}{12} \)

10. The value of the integral \( \int_{1}^{6} \frac{x^2 + 3x - 5}{x^2} \, dx \) is

(a) \( \frac{5}{6} + 3 \ln 6 \)
(b) \( \frac{5}{6} - 3 \ln 6 \)
(c) \( \frac{61}{6} + 3 \ln 6 \)
(d) \( \frac{17}{6} + 3 \ln 6 \)
(e) \( \frac{17}{6} - 3 \ln 6 \)
11. The value of the integral \[ \int_{0}^{\pi/4} \frac{1 + \sin \theta}{\cos^2 \theta} \, d\theta \] is

   (a) \( \sqrt{2} \)

   (b) 2

   (c) \( \frac{\sqrt{2}}{2} \)

   (d) \( \frac{3\sqrt{2}}{2} \)

   (e) \( \frac{\sqrt{2}}{3} \)

12. The area under the curve \( y = e^{\cos x} \sin x \) from 0 to \( \frac{\pi}{2} \) is

   (a) \( e - 1 \)

   (b) \( e + 1 \)

   (c) \( e^{-1} - 1 \)

   (d) \( e^{-1} + 1 \)

   (e) \( e + e^{-1} \)
13. The value of the integral $\int_1^e \frac{dx}{2x + x \ln x^3}$ is

(a) $\frac{1}{3} \ln \frac{5}{2}$

(b) $\ln \frac{5}{2}$

(c) $\frac{1}{3} \ln 5$

(d) $\frac{1}{3} \ln 10$

(e) $\frac{1}{3 \ln 5}$

14. The area of the region bounded by the curves $y = \ln x, \; x + y = 1,$ and $y = 1$ is

(a) $e - \frac{3}{2}$

(b) $e - \frac{2}{3}$

(c) $e - \frac{1}{2}$

(d) $\frac{e}{2} - \frac{1}{2}$

(e) $e - \frac{1}{4}$
15. The area of the region inside the circle \( y^2 + x^2 - 2x = 0 \)
and above the parabola \( y = x^2 \) is

(a) \( \int_0^1 (\sqrt{1 - (x - 1)^2} - x^2) \, dx \)

(b) \( \int_0^1 (\sqrt{1 + (x - 1)^2} - x^2) \, dx \)

(c) \( \int_0^1 (\sqrt{1 - (1 - x)^2} + x^2) \, dx \)

(d) \( \int_{-1}^1 (\sqrt{1 - (x - 1)^2} - x^2) \, dx \)

(e) \( \int_{-1}^1 (\sqrt{(x - 1)^2 + 1 - x^2}) \, dx \)

16. The area of the region enclosed by the graphs of the functions
\( y = x^3 - x \) and \( y = 3x \) is

(a) 8
(b) 0
(c) \( \frac{7}{2} \)
(d) 4
(e) 2
17. The volume of the solid obtained when the region bounded by \(y = e^x\), \(y = \frac{1}{x+1}\), \(x = 0\) and \(x = 1\) is rotated about the \(x\)-axis is

(a) \(\frac{\pi}{2}(e^2 - 2)\)

(b) \(\pi(e^2 - 2)\)

(c) \(\pi\left(e^2 - \frac{1}{2}\right)\)

(d) \(\frac{\pi}{2}(e - 1)\)

(e) \(\frac{\pi}{2}(e^2 - 1)\)

18. The base of a solid is bounded by the curves \(y = x^3\), \(y = 0\) and \(x = 1\). If the cross-sections of the solid perpendicular to the \(x\)-axis are squares, then the volume of the solid is

(a) \(\frac{1}{7}\)

(b) \(\frac{3}{4}\)

(c) \(\frac{1}{2}\)

(d) \(\frac{3}{7}\)

(e) 1
19. The volume of the solid generated when the region enclosed by \( y = x^3 \), \( y = 1 \), and \( x = 0 \) is revolved about the line \( y = 1 \) is equal to

- (a) \( 2\pi \int_0^1 (y^{1/3} - y^{1/3}) \ dy \)
- (b) \( 2\pi \int_0^1 (y^{4/3} - y^{1/3}) \ dy \)
- (c) \( 2\pi \int_0^1 (1 - y) \ y \ dy \)
- (d) \( 2\pi \int_0^1 (y - 1) \ y \ dy \)
- (e) \( 2\pi \int_0^1 (y - 1)^2 \ dy \)

20. The volume of the solid generated by revolving the region bounded by the curves \( y = \frac{1}{x} \), \( y = 0 \), \( x = 1 \) and \( x = 3 \) about the line \( x = 3 \) is

- (a) \( 2\pi(3 \ln 3 - 2) \)
- (b) \( \pi \ln 3 \)
- (c) \( 3\pi \ln 2 \)
- (d) \( 2 - \ln 3 \)
- (e) \( \pi(1 - \ln 3) \)
Check that this exam has 20 questions.

Important Instructions:

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2. Use HB 2.5 pencils only.

3. Use a good eraser. DO NOT use the erasers attached to the pencil.

4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.

5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.

6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.

7. When bubbling, make sure that the bubbled space is fully covered.

8. When erasing a bubble, make sure that you do not leave any trace of penciling.
1. If \( f(x) = \begin{cases} \frac{-1}{x} & \text{if } 2 \leq x \leq 3 \\ x - 4 & \text{if } 3 \leq x \leq 9 \\ 5 & \text{if } 9 \leq x \leq 10 \end{cases} \), then \( \int_{2}^{10} f(x) \, dx \) is 

(a) 16  
(b) 18  
(c) 19  
(d) 17  
(e) 15

2. Using four rectangles and midpoint approximation, the area under the graph of \( y = x^2 \) from 1 to 9 is approximately equal to 

(a) 180  
(b) 164  
(c) 84  
(d) 120  
(e) 240
3. If $f$ is an even function and $\int_{-2}^{2} f(x)dx = 4$ and $\int_{0}^{7} f(x)dx = 3$, then $\int_{-2}^{7} f(x)dx$ is

(a) 5
(b) -1
(c) 7
(d) 1
(e) 10

4. An expression as a limit for the area under the graph of the function $y = x \cos x$ on $[0, \frac{\pi}{2}]$ is

(a) $\lim_{n \to +\infty} \sum_{i=1}^{n} \frac{\pi i}{2n^2} \cos \left( \frac{\pi i}{2n} \right)$
(b) $\lim_{n \to +\infty} \sum_{i=1}^{n} \frac{\pi^2 i}{2n^2} \cos \left( \frac{\pi i}{2n} \right)$
(c) $\lim_{n \to +\infty} \sum_{i=1}^{n} \frac{\pi^2 i}{(2n)^2} \cos \left( \frac{\pi i}{2n} \right)$
(d) $\lim_{n \to +\infty} \sum_{i=1}^{n} \frac{\pi^2 i}{n^2} \cos \left( \frac{\pi i}{n} \right)$
(e) $\lim_{n \to +\infty} \sum_{i=1}^{n} \frac{\pi^2 i}{n} \cos \left( \frac{\pi i}{n} \right)$
5. The limit \( \lim_{n \to +\infty} \sum_{i=1}^{n} \frac{4}{n} \cosh \left( \frac{4i}{n} + 2 \right) \) can be interpreted as the area under the graph of the function

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(a) \( \sqrt{270} \)

(b) \( \sqrt{255} \)

(c) \( \sqrt{257} \)

(d) 15

(e) 16
7. Let \( f(x) = \int_{x^2}^{x^3} \tan^{-1} t \, dt \). Then \( f(1) + f'(1) \) is

(a) \( \frac{\pi}{2} \)

(b) 0

(c) \( \frac{\pi}{6} \)

(d) 2

(e) \( \frac{\pi}{4} \)

8. If the velocity of a particle moving along a straight line is given by \( v(t) = \frac{1}{2} - \cos t \), the distance traveled during the interval time \( \left[ 0, \frac{\pi}{2} \right] \) is

(a) \( \sqrt{3} - 1 - \frac{\pi}{12} \)

(b) \( \sqrt{3} + 1 - \frac{\pi}{12} \)

(c) \( \sqrt{3} - 1 + \frac{\pi}{12} \)

(d) \( \frac{\pi}{4} - 1 \)

(e) \( 1 - \frac{\pi}{4} \)
9. The value of the integral $\int_{1}^{6} \frac{x^2 + 3x - 5}{x^2} \, dx$ is

(a) $\frac{5}{6} - 3 \ln 6$

(b) $\frac{17}{6} + 3 \ln 6$

(c) $\frac{17}{6} - 3 \ln 6$

(d) $\frac{61}{6} + 3 \ln 6$

(e) $\frac{5}{6} + 3 \ln 6$

10. The value of the integral $\int_{-4}^{0} (2 + \sqrt{16 - x^2}) \, dx$ is

(a) $8 + 4\pi$

(b) 8

(c) $4\pi$

(d) $2 + 8\pi$

(e) $-8 + 8\pi$
11. The area under the curve \( y = e^{\cos x} \sin x \) from 0 to \( \frac{\pi}{2} \) is

(a) \( e + 1 \)

(b) \( e + e^{-1} \)

(c) \( e - 1 \)

(d) \( e^{-1} - 1 \)

(e) \( e^{-1} + 1 \)

12. The value of the integral \( \int_{1}^{e} \frac{dx}{2x + x \ln x^3} \) is

(a) \( \frac{1}{3} \ln \frac{5}{2} \)

(b) \( \frac{1}{3 \ln 5} \)

(c) \( \frac{1}{3} \ln 5 \)

(d) \( \ln \frac{5}{2} \)

(e) \( \frac{1}{3} \ln 10 \)
13. The value of the integral $\int_{0}^{\pi/4} \frac{1 + \sin \theta}{\cos^2 \theta} \, d\theta$ is

(a) $\frac{3\sqrt{2}}{2}$

(b) 2

(c) $\frac{\sqrt{2}}{3}$

(d) $\sqrt{2}$

(e) $\frac{\sqrt{2}}{2}$

14. The area of the region bounded by the curves $y = \ln x, \ x + y = 1,$ and $y = 1$ is

(a) $e - \frac{1}{4}$

(b) $e - \frac{1}{2}$

(c) $e - \frac{3}{2}$

(d) $e - \frac{2}{3}$

(e) $\frac{e}{2} - \frac{1}{2}$
15. The area of the region inside the circle $y^2 + x^2 - 2x = 0$ and above the parabola $y = x^2$ is

(a) $\int_{-1}^{1}(\sqrt{1 - (x - 1)^2} - x^2)dx$

(b) $\int_{0}^{1}(\sqrt{1 - (1 - x)^2} + x^2)dx$

(c) $\int_{-1}^{1}(\sqrt{(x - 1)^2 + 1} - x^2)dx$

(d) $\int_{0}^{1}(\sqrt{1 + (x - 1)^2} - x^2)dx$

(e) $\int_{0}^{1}(\sqrt{1 - (x - 1)^2} - x^2)dx$

16. The volume of the solid generated by revolving the region bounded by the curves $y = \frac{1}{x}, y = 0, x = 1$ and $x = 3$ about the line $x = 3$ is

(a) $\pi(1 - \ln 3)$

(b) $\pi \ln 3$

(c) $3\pi \ln 2$

(d) $2\pi(3 \ln 3 - 2)$

(e) $2 - \ln 3$
17. The area of the region enclosed by the graphs of the functions 
\( y = x^3 - x \) and \( y = 3x \) is

(a) 2
(b) \( \frac{7}{2} \)
(c) 8
(d) 4
(e) 0

18. The base of a solid is bounded by the curves \( y = x^3 \), \( y = 0 \) and \( x = 1 \). If the cross-sections of the solid perpendicular to the \( x \)-axis are squares, then the volume of the solid is

(a) 1
(b) \( \frac{3}{7} \)
(c) \( \frac{1}{2} \)
(d) \( \frac{3}{4} \)
(e) \( \frac{1}{7} \)
19. The volume of the solid generated when the region enclosed by \( y = x^3 \), \( y = 1 \), and \( x = 0 \) is revolved about the line \( y = 1 \) is equal to

(a) \( 2\pi \int_0^1 (y - 1)^2 \, dy \)

(b) \( 2\pi \int_0^1 (y^{4/3} - y^{1/3}) \, dy \)

(c) \( 2\pi \int_0^1 (1 - y) \, y \, dy \)

(d) \( 2\pi \int_0^1 (y - 1) \, y \, dy \)

(e) \( 2\pi \int_0^1 (y^{1/3} - y^{4/3}) \, dy \)

20. The volume of the solid obtained when the region bounded by \( y = e^x \), \( y = \frac{1}{x + 1} \), \( x = 0 \) and \( x = 1 \) is rotated about \( x \)-axis is

(a) \( \pi(e^2 - 2) \)

(b) \( \pi \left(e^2 - \frac{1}{2}\right) \)

(c) \( \frac{\pi}{2}(e^2 - 1) \)

(d) \( \frac{\pi}{2}(e - 1) \)

(e) \( \frac{\pi}{2}(e^2 - 2) \)
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ID: ______________________ Sec: ______________________

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7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.
1. Using four rectangles and midpoint approximation, the area under the graph of \( y = x^2 \) from 1 to 9 is approximately equal to

(a) 164
(b) 120
(c) 84
(d) 180
(e) 240

2. If \( f \) is an even function and \( \int_{-2}^{2} f(x)\,dx = 4 \) and \( \int_{0}^{7} f(x)\,dx = 3 \), then \( \int_{-2}^{7} f(x)\,dx \) is

(a) 1
(b) 7
(c) −1
(d) 10
(e) 5
3. If \( f(x) = \begin{cases} -1, & 2 \leq x \leq 3 \\ x - 4, & 3 \leq x \leq 9 \\ 5, & 9 \leq x \leq 10 \end{cases} \), then \( \int_{2}^{10} f(x) \, dx \) is

(a) 18  
(b) 17  
(c) 19  
(d) 16  
(e) 15

4. An expression as a limit for the area under the graph of the function \( y = x \cos x \) on \( [0, \frac{\pi}{2}] \) is

(a) \( \lim_{n \to +\infty} \sum_{i=1}^{n} \frac{\pi i}{2n^2} \cos \left( \frac{\pi i}{2n} \right) \) 

(b) \( \lim_{n \to +\infty} \sum_{i=1}^{n} \frac{\pi^2 i}{2n^2} \cos \left( \frac{\pi i}{2n} \right) \) 

(c) \( \lim_{n \to +\infty} \sum_{i=1}^{n} \frac{\pi^2 i}{(2n)^2} \cos \left( \frac{\pi i}{2n} \right) \) 

(d) \( \lim_{n \to +\infty} \sum_{i=1}^{1=n} \frac{\pi^2 i}{n} \cos \left( \frac{\pi i}{n} \right) \) 

(e) \( \lim_{n \to +\infty} \sum_{i=1}^{n} \frac{\pi^2 i}{n^2} \cos \left( \frac{\pi i}{n} \right) \)
5. The limit \( \lim_{n \to +\infty} \sum_{i=1}^{n} \frac{4}{n} \cosh \left( \frac{4i}{n} + 2 \right) \) can be interpreted as the area under the graph of the function

(a) \( y = \cosh x, \quad 0 \leq x \leq 4 \)

(b) \( y = \cosh \frac{x}{2}, \quad 0 \leq x \leq 4 \)

(c) \( y = \cosh(x + 2), \quad 2 \leq x \leq 4 \)

(d) \( y = \cosh(x + 2), \quad 0 \leq x \leq 4 \)

(e) \( y = 2 \cosh x, \quad 0 \leq x \leq 4 \)

6. The value of the integral \( \int_{-4}^{0} (2 + \sqrt{16 - x^2}) \, dx \) is

(a) \( 8 + 4\pi \)

(b) \( -8 + 8\pi \)

(c) \( 8 \)

(d) \( 2 + 8\pi \)

(e) \( 4\pi \)
7. Let \( f(x) = \int_{x^2}^{x^3} \tan^{-1} t \, dt \). Then \( f(1) + f'(1) \) is

(a) 0
(b) 2
(c) \( \frac{\pi}{4} \)
(d) \( \frac{\pi}{6} \)
(e) \( \frac{\pi}{2} \)

8. If the velocity of a particle moving along a straight line is given by \( v(t) = \frac{1}{2} - \cos t \), the distance traveled during the interval time \( [0, \frac{\pi}{2}] \) is

(a) \( \sqrt{3} + 1 - \frac{\pi}{12} \)
(b) \( 1 - \frac{\pi}{4} \)
(c) \( \sqrt{3} - 1 - \frac{\pi}{12} \)
(d) \( \frac{\pi}{4} - 1 \)
(e) \( \sqrt{3} - 1 + \frac{\pi}{12} \)
9. The value of the integral \( \int_1^6 \frac{x^2 + 3x - 5}{x^2} \, dx \) is

(a) \( \frac{17}{6} + 3 \ln 6 \)

(b) \( \frac{17}{6} - 3 \ln 6 \)

(c) \( \frac{5}{6} - 3 \ln 6 \)

(d) \( \frac{5}{6} + 3 \ln 6 \)

(e) \( \frac{61}{6} + 3 \ln 6 \)

10. Let \( F(x) = \int_1^x f(t) \, dt \) where \( f(t) = \int_1^t \frac{\sqrt{1 + u^4}}{u} \, du \). Then \( F''(2) \) is

(a) \( \sqrt{255} \)

(b) \( \sqrt{257} \)

(c) 16

(d) \( \sqrt{270} \)

(e) 15
11. The value of the integral \( \int_1^e \frac{dx}{2x + x \ln x^3} \) is

(a) \( \frac{1}{3 \ln 5} \)

(b) \( \frac{1}{3} \ln \frac{5}{2} \)

(c) \( \frac{1}{3} \ln 10 \)

(d) \( \ln \frac{5}{2} \)

(e) \( \frac{1}{3} \ln 5 \)

12. The area of the region bounded by the curves \( y = \ln x, \ x + y = 1, \) and \( y = 1 \) is

(a) \( e - \frac{1}{2} \)

(b) \( e - \frac{3}{2} \)

(c) \( e - \frac{1}{4} \)

(d) \( \frac{e}{2} - \frac{1}{2} \)

(e) \( e - \frac{2}{3} \)
13. The area of the region inside the circle $y^2 + x^2 - 2x = 0$ and above the parabola $y = x^2$ is

(a) $\int_{-1}^{1} (\sqrt{1 - (x - 1)^2} - x^2)dx$

(b) $\int_{-1}^{1} (\sqrt{(x - 1)^2 + 1} - x^2)dx$

(c) $\int_{0}^{1} (\sqrt{1 - (x - 1)^2} - x^2)dx$

(d) $\int_{0}^{1} (\sqrt{1 + (x - 1)^2} - x^2)dx$

(e) $\int_{0}^{1} (\sqrt{1 - (1 - x)^2} + x^2)dx$

14. The area under the curve $y = e^{\cos x} \sin x$ from 0 to $\frac{\pi}{2}$ is

(a) $e^{-1} - 1$

(b) $e + e^{-1}$

(c) $e^{-1} + 1$

(d) $e - 1$

(e) $e + 1$
15. The value of the integral \( \int_{0}^{\pi/4} \frac{1 + \sin \theta}{\cos^2 \theta} \, d\theta \) is

(a) 2

(b) \( \frac{\sqrt{2}}{2} \)

(c) \( \frac{\sqrt{2}}{3} \)

(d) \( \frac{3\sqrt{2}}{2} \)

(e) \( \sqrt{2} \)

16. The volume of the solid generated by revolving the region bounded by the curves \( y = \frac{1}{x} \), \( y = 0 \), \( x = 1 \) and \( x = 3 \) about the line \( x = 3 \) is

(a) \( 2\pi(3 \ln 3 - 2) \)

(b) \( 3\pi \ln 2 \)

(c) \( \pi \ln 3 \)

(d) \( \pi(1 - \ln 3) \)

(e) \( 2 - \ln 3 \)
17. The base of a solid is bounded by the curves $y = x^3$, $y = 0$ and $x = 1$. If the cross-sections of the solid perpendicular to the $x$-axis are squares, then the volume of the solid is

(a) $1$
(b) $\frac{3}{7}$
(c) $\frac{1}{2}$
(d) $\frac{1}{7}$
(e) $\frac{3}{4}$

18. The volume of the solid obtained when the region bounded by $y = e^x$, $y = \frac{1}{x+1}$, $x = 0$ and $x = 1$ is rotated about $x$-axis is

(a) $\frac{\pi}{2}(e - 1)$
(b) $\frac{\pi}{2}(e^2 - 2)$
(c) $\pi(e^2 - 2)$
(d) $\pi \left( e^2 - \frac{1}{2} \right)$
(e) $\frac{\pi}{2}(e^2 - 1)$
19. The volume of the solid generated when the region enclosed by \( y = x^3 \), \( y = 1 \), and \( x = 0 \) is revolved about the line \( y = 1 \) is equal to

(a) \( 2\pi \int_0^1 (y^{1/3} - y^{4/3}) \, dy \)
(b) \( 2\pi \int_0^1 (y - 1)^2 \, dy \)
(c) \( 2\pi \int_0^1 (1 - y) \, y \, dy \)
(d) \( 2\pi \int_0^1 (y^{4/3} - y^{1/3}) \, dy \)
(e) \( 2\pi \int_0^1 (y - 1) \, y \, dy \)

20. The area of the region enclosed by the graphs of the functions \( y = x^3 - x \) and \( y = 3x \) is

(a) \( \frac{7}{2} \)
(b) 4
(c) 8
(d) 0
(e) 2
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Check that this exam has 20 questions.

Important Instructions:

1. All types of calculators, pagers or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.
1. An expression as a limit for the area under the graph of the function \( y = x \cos x \) on \( [0, \frac{\pi}{2}] \) is

(a) \( \lim_{n \to +\infty} \sum_{i=1}^{\frac{\pi}{2}} \frac{\pi i}{n} \cos \left( \frac{\pi i}{n} \right) \)

(b) \( \lim_{n \to +\infty} \sum_{i=1}^{\frac{\pi}{2}} \frac{\pi i}{(2n)^2} \cos \left( \frac{\pi i}{2n} \right) \)

(c) \( \lim_{n \to +\infty} \sum_{i=1}^{\frac{\pi}{2}} \frac{\pi i}{2n^2} \cos \left( \frac{\pi i}{2n} \right) \)

(d) \( \lim_{n \to +\infty} \sum_{i=1}^{\frac{\pi}{2}} \frac{\pi i}{2n^2} \cos \left( \frac{\pi i}{2n} \right) \)

(e) \( \lim_{n \to +\infty} \sum_{i=1}^{\frac{\pi}{2}} \frac{\pi^2 i}{n^2} \cos \left( \frac{\pi i}{n} \right) \)

2. Using four rectangles and midpoint approximation, the area under the graph of \( y = x^2 \) from 1 to 9 is approximately equal to

(a) 180

(b) 120

(c) 164

(d) 240

(e) 84
3. If \( f \) is an even function and \( \int_{-2}^{2} f(x) \, dx = 4 \) and \( \int_{0}^{7} f(x) \, dx = 3 \), then \( \int_{-2}^{7} f(x) \, dx \) is

(a) 7
(b) \(-1\)
(c) 10
(d) 5
(e) 1

4. The limit \( \lim_{n \to +\infty} \sum_{i=1}^{n} \frac{4}{n} \cosh \left( \frac{4i}{n} + 2 \right) \) can be interpreted as the area under the graph of the function

(a) \( y = \cosh(x + 2), \quad 0 \leq x \leq 4 \)
(b) \( y = \cosh(x + 2), \quad 2 \leq x \leq 4 \)
(c) \( y = \cosh x, \quad 0 \leq x \leq 4 \)
(d) \( y = 2 \cosh x, \quad 0 \leq x \leq 4 \)
(e) \( y = \cosh \frac{x}{2}, \quad 0 \leq x \leq 4 \)
5. If \( f(x) = \begin{cases} 
-1, & 2 \leq x \leq 3 \\
x - 4, & 3 \leq x \leq 9 \\
5, & 9 \leq x \leq 10 
\end{cases} \), then \( \int_{2}^{10} f(x) \, dx \) is 

(a) 17  
(b) 18  
(c) 16  
(d) 15  
(e) 19

6. The value of the integral \( \int_{1}^{6} \frac{x^2 + 3x - 5}{x^2} \, dx \) is 

(a) \( \frac{17}{6} + 3 \ln 6 \)  
(b) \( \frac{5}{6} - 3 \ln 6 \)  
(c) \( \frac{61}{6} + 3 \ln 6 \)  
(d) \( \frac{5}{6} + 3 \ln 6 \)  
(e) \( \frac{17}{6} - 3 \ln 6 \)
7. Let \( F(x) = \int_1^x f(t) \, dt \) where \( f(t) = \int_1^t \frac{\sqrt{1 + u^4}}{u} \, du \). Then \( F''(2) \) is

(a) \( \sqrt{270} \)
(b) 15
(c) \( \sqrt{255} \)
(d) \( \sqrt{257} \)
(e) 16

8. Let \( f(x) = \int_{x^2}^{x^3} \tan^{-1} t \, dt \). Then \( f(1) + f'(1) \) is

(a) \( \frac{\pi}{2} \)
(b) 2
(c) 0
(d) \( \frac{\pi}{6} \)
(e) \( \frac{\pi}{4} \)
9. If the velocity of a particle moving along a straight line is given by \( v(t) = \frac{1}{2} - \cos t \), the distance traveled during the interval time \([0, \frac{\pi}{2}]\) is

(a) \( \frac{\pi}{4} - 1 \)

(b) \( \sqrt{3} - 1 + \frac{\pi}{12} \)

(c) \( 1 - \frac{\pi}{4} \)

(d) \( \sqrt{3} - 1 - \frac{\pi}{12} \)

(e) \( \sqrt{3} + 1 - \frac{\pi}{12} \)

10. The value of the integral \( \int_{-4}^{0} (2 + \sqrt{16 - x^2}) \, dx \) is

(a) 8

(b) \( 2 + 8\pi \)

(c) \( -8 + 8\pi \)

(d) \( 4\pi \)

(e) \( 8 + 4\pi \)
11. The area of the region inside the circle $y^2 + x^2 - 2x = 0$ and above the parabola $y = x^2$ is

(a) $\int_{-1}^{1} (\sqrt{1 - (x - 1)^2} - x^2) dx$

(b) $\int_{-1}^{1} (\sqrt{(x - 1)^2 + 1} - x^2) dx$

(c) $\int_{0}^{1} (\sqrt{1 + (x - 1)^2} - x^2) dx$

(d) $\int_{0}^{1} (\sqrt{1 - (x - 1)^2} - x^2) dx$

(e) $\int_{0}^{1} (\sqrt{1 - (1-x)^2} + x^2) dx$

12. The value of the integral $\int_{1}^{e} \frac{dx}{2x + x \ln x^3}$ is

(a) $\frac{1}{3} \ln 10$

(b) $\frac{1}{3 \ln 5}$

(c) $\frac{1}{3} \ln 5$

(d) $\frac{1}{3} \ln \frac{5}{2}$

(e) $\ln \frac{5}{2}$
13. The value of the integral \( \int_{0}^{\pi/4} \frac{1 + \sin \theta}{\cos^2 \theta} \, d\theta \) is

(a) \( 2 \)

(b) \( \sqrt{2} \)

(c) \( \frac{\sqrt{2}}{2} \)

(d) \( \frac{\sqrt{2}}{3} \)

(e) \( \frac{3\sqrt{2}}{2} \)

14. The area of the region bounded by the curves \( y = \ln x, \ x + y = 1, \) and \( y = 1 \) is

(a) \( e - \frac{1}{4} \)

(b) \( e - \frac{3}{2} \)

(c) \( \frac{e}{2} - \frac{1}{2} \)

(d) \( e - \frac{2}{3} \)

(e) \( e - \frac{1}{2} \)
15. The area under the curve $y = e^{\cos x} \sin x$ from 0 to $\frac{\pi}{2}$ is

(a) $e^{-1} + 1$

(b) $e - 1$

(c) $e^{-1} - 1$

(d) $e + 1$

(e) $e + e^{-1}$

16. The volume of the solid generated when the region enclosed by $y = x^3$, $y = 1$, and $x = 0$ is revolved about the line $y = 1$ is equal to

(a) $2\pi \int_0^1 (y^{4/3} - y^{1/3}) \, dy$

(b) $2\pi \int_0^1 (y - 1)^2 \, dy$

(c) $2\pi \int_0^1 (1 - y) \, y \, dy$

(d) $2\pi \int_0^1 (y - 1) \, y \, dy$

(e) $2\pi \int_0^1 (y^{1/3} - y^{4/3}) \, dy$
17. The volume of the solid generated by revolving the region bounded by the curves \( y = \frac{1}{x} \), \( y = 0 \), \( x = 1 \) and \( x = 3 \) about the line \( x = 3 \) is

(a) \( 3\pi \ln 2 \)
(b) \( 2 - \ln 3 \)
(c) \( 2\pi(3\ln 3 - 2) \)
(d) \( \pi(1 - \ln 3) \)
(e) \( \pi \ln 3 \)

18. The volume of the solid obtained when the region bounded by \( y = e^x \), \( y = \frac{1}{x + 1} \), \( x = 0 \) and \( x = 1 \) is rotated about \( x \)-axis is

(a) \( \pi \left( e^2 - \frac{1}{2} \right) \)
(b) \( \pi(e^2 - 2) \)
(c) \( \frac{\pi}{2}(e^2 - 2) \)
(d) \( \frac{\pi}{2}(e - 1) \)
(e) \( \frac{\pi}{2}(e^2 - 1) \)
19. The base of a solid is bounded by the curves $y = x^3$, $y = 0$ and $x = 1$. If the cross-sections of the solid perpendicular to the $x$-axis are squares, then the volume of the solid is

(a) 1
(b) $\frac{3}{7}$
(c) $\frac{1}{2}$
(d) $\frac{3}{4}$
(e) $\frac{1}{7}$

20. The area of the region enclosed by the graphs of the functions $y = x^3 - x$ and $y = 3x$ is

(a) 4
(b) $\frac{7}{2}$
(c) 0
(d) 2
(e) 8
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Check that this exam has 20 questions.

Important Instructions:

1. All types of calculators, pagers or mobile phones are NOT allowed during the examination.

2. Use HB 2.5 pencils only.

3. Use a good eraser. DO NOT use the erasers attached to the pencil.

4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.

5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.

6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.

7. When bubbling, make sure that the bubbled space is fully covered.

8. When erasing a bubble, make sure that you do not leave any trace of penciling.
1. The limit \( \lim_{n \to \infty} \sum_{i=1}^{n} \frac{4}{n} \cosh \left( \frac{4i}{n} + 2 \right) \) can be interpreted as the area under the graph of the function

(a) \( y = \cosh(x + 2), \ 0 \leq x \leq 4 \)

(b) \( y = \cosh \frac{x}{2}, \ 0 \leq x \leq 4 \)

(c) \( y = \cosh x, \ 0 \leq x \leq 4 \)

(d) \( y = 2 \cosh x, \ 0 \leq x \leq 4 \)

(e) \( y = \cosh(x + 2), \ 2 \leq x \leq 4 \)

2. If \( f \) is an even function and \( \int_{-2}^{2} f(x)dx = 4 \) and \( \int_{0}^{7} f(x)dx = 3 \), then \( \int_{-2}^{7} f(x)dx \) is

(a) 7

(b) 10

(c) −1

(d) 1

(e) 5
3. If \( f(x) = \begin{cases} -1, & 2 \leq x \leq 3 \\ x - 4, & 3 \leq x \leq 9 \\ 5, & 9 \leq x \leq 10 \end{cases} \), then \( \int_{2}^{10} f(x) \, dx \) is

(a) 19
(b) 15
(c) 16
(d) 17
(e) 18

4. An expression as a limit for the area under the graph of the function \( y = x \cos x \) on \( [0, \frac{\pi}{2}] \) is

(a) \( \lim_{n \to +\infty} \sum_{i=1}^{n} \frac{\pi^2 i}{n^2} \cos \left( \frac{\pi i}{n} \right) \)
(b) \( \lim_{n \to +\infty} \sum_{i=1}^{n} \frac{\pi i}{2n^2} \cos \left( \frac{\pi i}{2n} \right) \)
(c) \( \lim_{n \to +\infty} \sum_{i=1}^{n} \frac{\pi^2 i^2}{2n^2} \cos \left( \frac{\pi i}{2n} \right) \)
(d) \( \lim_{n \to +\infty} \sum_{i=1}^{n} \frac{\pi^2 i^2}{n^2} \cos \left( \frac{\pi i}{n} \right) \)
(e) \( \lim_{n \to +\infty} \sum_{i=1}^{n} \frac{\pi^2 i}{(2n)^2} \cos \left( \frac{\pi i}{2n} \right) \)
5. Using four rectangles and midpoint approximation, the area under the graph of \( y = x^2 \) from 1 to 9 is approximately equal to

(a) 180
(b) 84
(c) 120
(d) 240
(e) 164

6. If the velocity of a particle moving along a straight line is given by \( v(t) = \frac{1}{2} - \cos t \), the distance traveled during the interval time \([0, \frac{\pi}{2}]\) is

(a) \( \frac{\pi}{4} - 1 \)
(b) \( \sqrt{3} + 1 - \frac{\pi}{12} \)
(c) \( \sqrt{3} - 1 + \frac{\pi}{12} \)
(d) \( \sqrt{3} - 1 - \frac{\pi}{12} \)
(e) \( 1 - \frac{\pi}{4} \)
7. The value of the integral \( \int_{-4}^{0} (2 + \sqrt{16 - x^2}) \, dx \) is

(a) \( 4\pi \)

(b) \( 2 + 8\pi \)

(c) \( 8 + 4\pi \)

(d) \( -8 + 8\pi \)

(e) \( 8 \)

8. Let \( F(x) = \int_{1}^{x} f(t) \, dt \) where \( f(t) = \int_{1}^{t^2} \frac{\sqrt{1 + u^4}}{u} \, du \). Then \( F''(2) \) is

(a) \( 15 \)

(b) \( \sqrt{255} \)

(c) \( \sqrt{270} \)

(d) \( 16 \)

(e) \( \sqrt{257} \)
9. Let \( f(x) = \int_{x^2}^{x^3} \tan^{-1} t \, dt \). Then \( f(1) + f'(1) \) is

(a) \( \frac{\pi}{6} \)

(b) \( \frac{\pi}{2} \)

(c) 2

(d) \( \frac{\pi}{4} \)

(e) 0

10. The value of the integral \( \int_1^6 \frac{x^2 + 3x - 5}{x^2} \, dx \) is

(a) \( \frac{17}{6} + 3 \ln 6 \)

(b) \( \frac{5}{6} - 3 \ln 6 \)

(c) \( \frac{17}{6} - 3 \ln 6 \)

(d) \( \frac{5}{6} + 3 \ln 6 \)

(e) \( \frac{61}{6} + 3 \ln 6 \)
11. The value of the integral $\int_1^e \frac{dx}{2x + x \ln x^3}$ is

(a) $\frac{1}{3} \ln \frac{5}{2}$

(b) $\frac{1}{3} \ln 5$

(c) $\ln \frac{5}{2}$

(d) $\frac{1}{3} \ln 5$

(e) $\frac{1}{3} \ln 10$

12. The area of the region inside the circle $y^2 + x^2 - 2x = 0$ and above the parabola $y = x^2$ is

(a) $\int_{-1}^{1} (\sqrt{(x - 1)^2 + 1} - x^2) \, dx$

(b) $\int_{-1}^{1} (\sqrt{1 - (x - 1)^2} - x^2) \, dx$

(c) $\int_{0}^{1} (\sqrt{1 - (x - 1)^2} - x^2) \, dx$

(d) $\int_{0}^{1} (\sqrt{1 + (x - 1)^2} - x^2) \, dx$

(e) $\int_{0}^{1} (\sqrt{1 - (1 - x)^2} + x^2) \, dx$
13. The value of the integral \( \int_{0}^{\pi/4} \frac{1 + \sin \theta}{\cos^2 \theta} \, d\theta \) is

(a) \( \frac{3\sqrt{2}}{2} \)

(b) \( \frac{\sqrt{2}}{3} \)

(c) \( \frac{\sqrt{2}}{2} \)

(d) \( \sqrt{2} \)

(e) 2

14. The area of the region bounded by the curves \( y = \ln x, \ x + y = 1, \) and \( y = 1 \) is

(a) \( e - \frac{1}{4} \)

(b) \( e - \frac{3}{2} \)

(c) \( e - \frac{2}{3} \)

(d) \( e - \frac{1}{2} \)

(e) \( \frac{e}{2} - \frac{1}{2} \)
15. The area under the curve $y = e^{\cos x} \sin x$ from 0 to $\frac{\pi}{2}$ is

(a) $e + e^{-1}$

(b) $e^{-1} - 1$

(c) $e^{-1} + 1$

(d) $e - 1$

(e) $e + 1$

16. The volume of the solid generated by revolving the region bounded by the curves $y = \frac{1}{x}, \ y = 0, \ x = 1$ and $x = 3$ about the line $x = 3$ is

(a) $\pi \ln 3$

(b) $2 - \ln 3$

(c) $2\pi(3\ln 3 - 2)$

(d) $3\pi \ln 2$

(e) $\pi(1 - \ln 3)$
17. The volume of the solid obtained when the region bounded by \( y = e^x \), \( y = \frac{1}{x + 1} \), \( x = 0 \) and \( x = 1 \) is rotated about \( x \)-axis is

(a) \( \pi \left( e^2 - \frac{1}{2} \right) \)

(b) \( \frac{\pi}{2} (e - 1) \)

(c) \( \frac{\pi}{2} (e^2 - 2) \)

(d) \( \frac{\pi}{2} (e^2 - 1) \)

(e) \( \pi (e^2 - 2) \)

18. The base of a solid is bounded by the curves \( y = x^3 \), \( y = 0 \) and \( x = 1 \). If the cross-sections of the solid perpendicular to the \( x \)-axis are squares, then the volume of the solid is

(a) \( \frac{3}{4} \)

(b) \( \frac{1}{2} \)

(c) \( \frac{3}{7} \)

(d) \( \frac{1}{7} \)

(e) 1
19. The volume of the solid generated when the region enclosed by $y = x^3$, $y = 1$, and $x = 0$ is revolved about the line $y = 1$ is equal to

(a) $2\pi \int_0^1 (y^{1/3} - y^{1/3}) \, dy$

(b) $2\pi \int_0^1 (y - 1) \, y \, dy$

(c) $2\pi \int_0^1 (y^{4/3} - y^{1/3}) \, dy$

(d) $2\pi \int_0^1 (1 - y) \, y \, dy$

(e) $2\pi \int_0^1 (y - 1)^2 \, dy$

20. The area of the region enclosed by the graphs of the functions $y = x^3 - x$ and $y = 3x$ is

(a) 4

(b) 8

(c) 0

(d) 2

(e) $\frac{7}{2}$
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