### Problem No. # | Grade | Maximum Points
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1 | | 6
2a | | 6
2b | | 6
3 | | 12
4 | | 8
5 | | 8
6 | | 9
7 | | 14
8 | | 12
9 | | 9
10 | | 10
**Total** | | 100
1. (6 points) Find all positive numbers $b$ such that the average value of the function $f(x) = 2 + 6x - 3x^2$ on the interval $[0, b]$ is equal to 3.
2. (a) (6 points) Evaluate \( \int x(x + 1)e^x \, dx \).

(b) (6 points) Evaluate \( \int x \tan^{-1} x \, dx \).
3. (12 points) Evaluate \[ \int \sin^2 x \cos^4 x \, dx. \]
4. (8 points) Evaluate \[ \int \frac{\sin^3 \theta \, d\theta}{\cos^6 \theta}. \]
5. (8 points) Evaluate \( \int \frac{dx}{(4-x^2)^{3/2}} \).
6. (9 points) Evaluate \( \int \frac{x^5 + 2}{x^2 - 1} \, dx \).
7. (14 points) Evaluate \( \int \frac{dx}{x^3 + 1} \).
8. (12 points) Evaluate \( \int \frac{\sin 2x}{1 + \cos^4 x} \, dx \).
9. (9 points) Determine whether the integral \( \int_1^3 \frac{1}{\sqrt{x-1}} \, dx \) is convergent or divergent.
10. (10 points) Determine whether the improper integral \( \int_{1}^{+\infty} \frac{5 - 2\sin x}{\sqrt{x^3}} \, dx \) is convergent or divergent.