Math 102
Final Exam
Second
Saturday, June 11, 2011

EXAM COVER

Number of versions: 4
Number of questions: 28
Number of Answers: 5 per question

This exam was prepared using mcqs
For questions send an email to Dr. Ibrahim Al-Lehyani (iallehyani@kaau.edu.sa)
1. If \( F(x) = \int_x^{x^2} \frac{dt}{\sqrt{1-t^2}} \), then \( F'(\frac{1}{2}) \) is

(a) \( \frac{4 - 2\sqrt{5}}{\sqrt{15}} \)

(b) \( \frac{4 + 2\sqrt{5}}{\sqrt{15}} \)

(c) \( \frac{4}{\sqrt{15}} - \frac{1}{\sqrt{3}} \)

(d) \( \frac{2 - 2\sqrt{5}}{\sqrt{15}} \)

(e) \( \frac{2 + 2\sqrt{5}}{\sqrt{15}} \)

2. The value of the integral \( \int_{0}^{\cos^{-1}(1/e)} (\tan x) \ln(\cos x) \, dx \) is

(a) \(-1/2\)

(b) \(1/2\)

(c) \(-\sqrt{2}/2\)

(d) \(e/2\)

(e) \(-e/2\)
3. The value of the integral \( \int_{-1}^{0} x^2\sqrt{1+x} \, dx \) is

(a) \( \frac{16}{105} \)

(b) \( \frac{19}{105} \)

(c) \( \frac{8}{105} \)

(d) \( \frac{22}{105} \)

(e) \( \frac{102}{105} \)

4. The value of the integral \( \int_{0}^{1} x\sqrt{1-x^4} \, dx \) is

(a) \( \frac{\pi}{8} \)

(b) \( \frac{\pi}{16} \)

(c) \( \frac{\pi}{3} \)

(d) \( \frac{\pi}{6} \)

(e) \( \frac{\pi}{2} \)
5. The area of the region bounded by the curves $4x + y^2 = 12$ and $x = y$ is

(a) $\frac{64}{3}$

(b) 21

(c) $\frac{44}{3}$

(d) $\frac{52}{3}$

(e) 17

6. The volume of the solid obtained by rotating the region bounded by the curves $y = e^{x^2}$, $y = e$ and $y = 0$ (in the first quadrant) about $y$-axis is

(a) $\pi$

(b) $2\pi$

(c) $e^2\pi$

(d) $\pi - e$

(e) $\pi - e^2$
7. The volume of the solid obtained by rotating the region bounded by the curves $x^2 - y^2 = 1$ and $x = 3$ about the line $x = -2$ is given by

(a) $\int_{-2\sqrt{2}}^{2\sqrt{2}} \pi [25 - (2 + \sqrt{1 + y^2})^2] dy$

(b) $\int_{0}^{2\sqrt{2}} \pi [25 - (2 + y)^2] dy$

(c) $\int_{-3}^{3} \pi [25 - (2 + \sqrt{x^2 - 1})^2] dx$

(d) $\int_{-2\sqrt{2}}^{2\sqrt{2}} \pi [9 - (1 + y^2)] dy$

(e) $\int_{-3}^{3} \pi [9 - (x^2 - 1)] dx$

8. If $f(x) = f'(x) + 3$, $f(0) = 1$ and $f(1) = 4$, then the value of the integral $\int_{0}^{1} e^x f'(x) dx$ is

(a) $\frac{e + 2}{2}$

(b) $\frac{e - 2}{2}$

(c) $\frac{2e - 1}{2}$

(d) $\frac{e + 1}{2}$

(e) $\frac{e - 1}{2}$
9. The value of the integral $\int_0^{\pi/4} \tan^4 x \sec^4 x \, dx$ is

(a) $\frac{12}{35}$

(b) $\frac{2}{35}$

(c) $-\frac{2}{35}$

(d) $\frac{1}{5}$

(e) $\frac{5}{12}$

10. The value of the integral $\int_0^1 x^3 \sqrt{1 + x^2} \, dx$ is

(a) $\frac{2}{15} \sqrt{2} + \frac{2}{15}$

(b) $\frac{\sqrt{2}}{3} - \frac{1}{15}$

(c) $\sqrt{2} - 1$

(d) $\frac{1}{3} - \sqrt{2}$

(e) $\frac{1}{6} - \frac{\sqrt{2}}{2}$
11. The value of the integral \( \int_0^1 \frac{3x + 2}{x^2 - 4} \, dx \) is

(a) \( \ln 3 - 3 \ln 2 \)
(b) \( \ln 2 - 2 \ln 3 \)
(c) \( 3 \ln 2 - \ln 3 \)
(d) \( 2 \ln 3 - \ln 2 \)
(e) \( \ln 3 - \ln 2 \)

12. The improper integral \( \int_1^\infty \frac{e^{-2\sqrt{x}}}{\sqrt{x}} \, dx \) is

(a) Convergent and its value is \( e^{-2} \)
(b) Convergent and its value is \( e^{+2} \)
(c) Convergent and its value is \( -e^{-2} \)
(d) Convergent and its value is 0
(e) Divergent
13. The sequence \( \left\{ \frac{\ln n}{n^2} \right\}_{n \geq 3} \) is

(a) Decreasing and convergent
(b) Decreasing and divergent
(c) Increasing and convergent
(d) Increasing and divergent
(e) Neither increasing, nor decreasing and convergent

14. The series \( \sum_{n=0}^{\infty} \frac{e^{1-2n}}{(\sqrt{2})^{2-2n}} \) is

(a) Convergent and its sum is \( \frac{e^3}{2e^2 - 4} \)
(b) Convergent and its sum is \( \frac{1}{e^3} \)
(c) Convergent and its sum is \( \frac{e}{2} \)
(d) Convergent and its sum is \( \frac{e^2}{e^2 - 2} \)
(e) Divergent
15. For which values of $p$, the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ is convergent

(a) $p > 1$

(b) $p < 1$

(c) $p > e$

(d) converges for all $p$

(e) diverges for all $p$

16. The series $\sum_{n=1}^{+\infty} \frac{\cos(n\pi)}{n}$ is

(a) Conditionally convergent

(b) Divergent

(c) Absolutely convergent

(d) Divergent by Ratio Test

(e) Divergent by Comparison Test
17. If the sum of the first \( n \) terms of a series \( \sum_{n=1}^{\infty} a_n \) is given by

\[ S_n = \frac{2n}{n+1}, \text{ then} \]

(a) \( a_n = \frac{2}{n^2 + n} \)

(b) \( a_n = \frac{1}{n^2 + n} \)

(c) \( a_n = \frac{4}{n^2 + n} \)

(d) \( a_n = \frac{2}{n^2 + 2n} \)

(e) \( a_n = \frac{2}{n^2 + 1} \)

18. The minimum number of terms needed to estimate the sum of the series \( \sum_{n=1}^{\infty} \frac{(-1)^n}{(n+1)^4} \) within 0.0001 (error) is

(a) \( n = 10 \)

(b) \( n = 13 \)

(c) \( n = 12 \)

(d) \( n = 11 \)

(e) \( n = 8 \)
19. The series \( \sum_{n=1}^{+\infty} \left( \frac{\cos(n^2)}{n^2} \right)^n \) is

(a) Convergent
(b) Divergent by Divergence Test
(c) Divergent by Ratio Test
(d) Conditionally convergent
(e) Divergent by Comparison Test

20. The series \( \sum_{n=0}^{+\infty} \frac{2^n \sin\left(\frac{n\pi}{2}\right)}{n!} \) is

(a) Convergent by Alternating Series Test
(b) Divergent by Ratio Test
(c) Divergent by Divergence Test
(d) Divergent by Comparison Test
(e) Convergent by Limit Comparison Test
21. The radius of convergence of the series \( \sum_{n=0}^{\infty} \frac{(n!)^2 x^n}{(2n)!} \) is

(a) 4
(b) \( e^2 \)
(c) 0
(d) \( k \)
(e) 1

22. The interval of convergence of the series \( \sum_{n=1}^{\infty} (-1)^n \frac{(x + 2)^n}{n4^n} \) is

(a) \((-6, 2]\)
(b) \((-6, 2)\)
(c) \([-6, 2)\)
(d) \([-6, 2]\)
(e) \([-2, 2]\)
23. A power series representing the function \( f(x) = \frac{3x^3}{(x-3)^2} \) is

\[
\text{Hint: You may consider the function: } \frac{d}{dx} \left( \frac{1}{3-x} \right) = \frac{1}{(3-x)^2}
\]

(a) \( \sum_{n=1}^{\infty} \frac{n\cdot x^{n+2}}{3^n} \)

(b) \( \sum_{n=1}^{\infty} \frac{x^{n+3}}{3^n} \)

(c) \( \sum_{n=1}^{\infty} \frac{n\cdot x^n}{3^{n+2}} \)

(d) \( \sum_{n=1}^{\infty} \frac{n\cdot x^n}{3^n} \)

(e) \( \sum_{n=1}^{\infty} \frac{x^n}{3^n} \)

24. The value of the integral \( \int_0^{1/3} \frac{x^2}{1+x^7} \) dx is

(a) \( \sum_{n=0}^{\infty} \frac{(-1)^n}{(7n+3) \cdot 3^{7n+3}} \)

(b) \( \sum_{n=0}^{\infty} \frac{(-1)^n}{(7n+1) \cdot 3^{7n+1}} \)

(c) \( \sum_{n=0}^{\infty} \frac{(-1)^n}{7n \cdot (3)^{7n+1}} \)

(d) \( \sum_{n=0}^{\infty} \frac{(-1)^n}{(7n+2) \cdot 3^{7n+2}} \)

(e) \( \sum_{n=0}^{\infty} \frac{(-1)^n}{(7n+1) \cdot 3^{7n+2}} \)
25. For $x > 0$ (but close to 1), the sum of the series $\sum_{n=0}^{\infty} \frac{(-2)^n (\ln x)^n}{n!}$ is

(a) $\frac{1}{x^2}$
(b) $x^2$
(c) $e^x$
(d) $e^{x^2}$
(e) $e^{-2x}$

26. The first four terms of the Taylor series of the function $f(x) = \ln(3 + x)$ at $a = 1$ are:

(a) $\ln 4 + \frac{(x - 1)}{4} - \frac{(x - 1)^2}{32} + \frac{(x - 1)^3}{192}$
(b) $\ln 4 - \frac{(x - 1)}{4} + \frac{(x - 1)^2}{32} - \frac{(x - 1)^3}{192}$
(c) $\frac{(x - 1)}{4} - \frac{(x - 1)^2}{32} + \frac{(x - 1)^3}{192} - \frac{(x - 1)^4}{256}$
(d) $-\frac{(x - 1)}{4} + \frac{(x - 1)^2}{32} - \frac{(x - 1)^3}{192} + \frac{(x - 1)^4}{256}$
(e) $\ln 4 + \frac{(x - 1)}{4} + \frac{(x - 1)^2}{32} + \frac{(x - 1)^3}{192}$
27. The length of the curve \( y = \ln(\sec x), \ 0 \leq x \leq \frac{\pi}{4} \) is

(a) \( \ln(1 + \sqrt{2}) \)
(b) \( \ln(\sqrt{2}) \)
(c) \( 1 + \sqrt{2} \)
(d) \( \ln(\sqrt{2} + \sqrt{3}) \)
(e) \( \ln(2 + \sqrt{2}) \)

28. The areas of the surface of the solid obtained by rotating the curve \( y = 2\sqrt{x}, \ 8 \leq x \leq 15 \) about \( x \)-axis is

(a) \( \frac{296\pi}{3} \)
(b) \( \frac{148\pi}{3} \)
(c) \( \frac{74\pi}{3} \)
(d) \( \frac{48\pi}{3} \)
(e) \( \frac{96\pi}{3} \)
Check that this exam has 28 questions.

Important Instructions:

1. All types of calculators, pagers or mobile phones are NOT allowed during the examination.

2. Use HB 2.5 pencils only.

3. Use a good eraser. DO NOT use the erasers attached to the pencil.

4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.

5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.

6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.

7. When bubbling, make sure that the bubbled space is fully covered.

8. When erasing a bubble, make sure that you do not leave any trace of penciling.
1. The volume of the solid obtained by rotating the region bounded by the curves $x^2 - y^2 = 1$ and $x = 3$ about the line $x = -2$ is given by

(a) $\int_{-2\sqrt{2}}^{2\sqrt{2}} \pi [9 - (1 + y^2)] dy$

(b) $\int_{0}^{2\sqrt{2}} \pi [25 - (2 + y)^2] dy$

(c) $\int_{-3}^{3} \pi [25 - (2 + \sqrt{x^2 - 1})^2] dx$

(d) $\int_{-2\sqrt{2}}^{2\sqrt{2}} \pi [25 - \left(2 + \sqrt{1 + y^2}\right)^2] dy$

(e) $\int_{-3}^{3} \pi [9 - (x^2 - 1)] dx$

2. The interval of convergence of the series $\sum_{n=1}^{\infty} (-1)^n \frac{(x + 2)^n}{n4^n}$ is

(a) $(-6, 2)$

(b) $[-6, 2)$

(c) $[-6, 2]$  

(d) $[-2, 2]$  

(e) $(-6, 2]$
3. If the sum of the first $n$ terms of a series $\sum_{n=1}^{+\infty} a_n$ is given by
\[ S_n = \frac{2n}{n + 1}, \text{ then} \]

(a) $a_n = \frac{2}{n^2 + 2n}$
(b) $a_n = \frac{1}{n^2 + n}$
(c) $a_n = \frac{4}{n^2 + n}$
(d) $a_n = \frac{2}{n^2 + 1}$
(e) $a_n = \frac{2}{n^2 + n}$

4. The value of the integral $\int_{0}^{1/3} \frac{x^2 dx}{1 + x^7}$ is

(a) $\sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n + 1) \cdot 3^{7n+1}}$
(b) $\sum_{n=0}^{+\infty} \frac{(-1)^n}{7n \cdot (3)^{7n+1}}$
(c) $\sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n + 1) \cdot 3^{7n+2}}$
(d) $\sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n + 3) \cdot 3^{7n+3}}$
(e) $\sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n + 2) \cdot 3^{7n+2}}$
5. If \( f(x) = f'(x) + 3 \), \( f(0) = 1 \) and \( f(1) = 4 \), then the value of the integral \( \int_{0}^{1} e^x f'(x)\,dx \) is

(a) \( \frac{2e - 1}{2} \)
(b) \( \frac{e + 1}{2} \)
(c) \( \frac{e - 1}{2} \)
(d) \( \frac{e + 2}{2} \)
(e) \( \frac{e - 2}{2} \)

6. The areas of the surface of the solid obtained by rotating the curve \( y = 2\sqrt{x} \), \( 8 \leq x \leq 15 \) about \( x \)-axis is

(a) \( \frac{74\pi}{3} \)
(b) \( \frac{148\pi}{3} \)
(c) \( \frac{96\pi}{3} \)
(d) \( \frac{48\pi}{3} \)
(e) \( \frac{296\pi}{3} \)
7. For which values of $p$, the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ is convergent?

(a) diverges for all $p$
(b) $p > e$
(c) converges for all $p$
(d) $p > 1$
(e) $p < 1$

8. A power series representing the function $f(x) = \frac{3x^3}{(x-3)^2}$ is

\[ \text{Hint: You may consider the function: } \frac{d}{dx} \left( \frac{1}{3-x} \right) = \frac{1}{(3-x)^2} \]

(a) $\sum_{n=1}^{+\infty} \frac{nx^n}{3^{n+2}}$
(b) $\sum_{n=1}^{+\infty} \frac{x^{n+3}}{3^n}$
(c) $\sum_{n=1}^{+\infty} \frac{x^n}{3^n}$
(d) $\sum_{n=1}^{+\infty} \frac{nx^{n+2}}{3^n}$
(e) $\sum_{n=1}^{+\infty} \frac{nx^n}{3^n}$
9. The sequence \( \left\{ \frac{\ln n}{n^2} \right\}_{n \geq 3} \) is

(a) Increasing and convergent
(b) Decreasing and convergent
(c) Decreasing and divergent
(d) Neither increasing, nor decreasing and convergent
(e) Increasing and divergent

10. The series \( \sum_{n=0}^{\infty} \frac{e^{1-2n}}{(\sqrt{2})^{2n-2n}} \) is

(a) Divergent
(b) Convergent and its sum is \( \frac{e^2}{e^2 - 2} \)
(c) Convergent and its sum is \( \frac{e}{2} \)
(d) Convergent and its sum is \( \frac{1}{e^3} \)
(e) Convergent and its sum is \( \frac{e^3}{2e^2 - 4} \)
11. The volume of the solid obtained by rotating the region bounded by the curves \( y = e^{x^2} \), \( y = e \) and \( y = 0 \) (in the first quadrant) about \( y \)-axis is

(a) \( \pi \)

(b) \( 2\pi \)

(c) \( \pi - e \)

(d) \( e^2\pi \)

(e) \( \pi - e^2 \)

12. The value of the integral \( \int_0^1 x\sqrt{1 - x^4} \, dx \) is

(a) \( \frac{\pi}{3} \)

(b) \( \frac{\pi}{16} \)

(c) \( \frac{\pi}{2} \)

(d) \( \frac{\pi}{8} \)

(e) \( \frac{\pi}{6} \)
13. The value of the integral \( \int_{-1}^{0} x^2 \sqrt{1 + x} \, dx \) is

(a) \( \frac{102}{105} \)

(b) \( \frac{16}{105} \)

(c) \( \frac{8}{105} \)

(d) \( \frac{22}{105} \)

(e) \( \frac{19}{105} \)

14. The series \( \sum_{n=0}^{\infty} \frac{2^n \sin \left( \frac{n\pi}{2} \right)}{n!} \) is

(a) Convergent by Alternating Series Test

(b) Divergent by Divergence Test

(c) Divergent by Ratio Test

(d) Convergent by Limit Comparison Test

(e) Divergent by Comparison Test
15. The area of the region bounded by the curves $4x + y^2 = 12$ and $x = y$ is

(a) $\frac{64}{3}$

(b) $\frac{44}{3}$

(c) 17

(d) 21

(e) $\frac{52}{3}$

16. The length of the curve $y = \ln(\sec x)$, $0 \leq x \leq \frac{\pi}{4}$ is

(a) $\ln(1 + \sqrt{2})$

(b) $1 + \sqrt{2}$

(c) $\ln(\sqrt{2})$

(d) $\ln(\sqrt{2} + \sqrt{3})$

(e) $\ln(2 + \sqrt{2})$
17. The series \( \sum_{n=1}^{+\infty} \left( \frac{\cos(n^2)}{n^2} \right)^n \) is

(a) Divergent by Ratio Test

(b) Conditionally convergent

(c) Divergent by Divergence Test

(d) Convergent

(e) Divergent by Comparison Test

18. The value of the integral \( \int_0^{\cos^{-1}(1/e)} (\tan x) \ln(\cos x) dx \) is

(a) 1/2

(b) \(-e/2\)

(c) \(-1/2\)

(d) \(e/2\)

(e) \(-\sqrt{2}/2\)
19. The improper integral \( \int_{1}^{\infty} \frac{e^{-2\sqrt{x}}}{\sqrt{x}} \, dx \) is

(a) Convergent and its value is \(-e^{-2}\)

(b) Convergent and its value is \(e^{-2}\)

(c) Convergent and its value is \(e^{-2}\)

(d) Divergent

(e) Convergent and its value is 0

20. The value of the integral \( \int_{0}^{1} \frac{3x + 2}{x^2 - 4} \, dx \) is

(a) \(2 \ln 3 - \ln 2\)

(b) \(\ln 3 - \ln 2\)

(c) \(3 \ln 2 - \ln 3\)

(d) \(\ln 3 - 3 \ln 2\)

(e) \(\ln 2 - 2 \ln 3\)
21. The minimum number of terms needed to estimate the sum of the series \[ \sum_{n=1}^{\infty} \frac{(-1)^n}{(n + 1)^4} \] within 0.0001 (error) is

(a) \( n = 10 \)
(b) \( n = 8 \)
(c) \( n = 13 \)
(d) \( n = 12 \)
(e) \( n = 11 \)

22. If \( F(x) = \int_{1/2}^{x^2} \frac{dt}{\sqrt{1-t^2}} \), then \( F'(\frac{1}{2}) \) is

(a) \( \frac{4 + 2\sqrt{5}}{\sqrt{15}} \)
(b) \( \frac{4}{\sqrt{15}} - \frac{1}{\sqrt{3}} \)
(c) \( \frac{2 + 2\sqrt{5}}{\sqrt{15}} \)
(d) \( \frac{4 - 2\sqrt{5}}{\sqrt{15}} \)
(e) \( \frac{2 - 2\sqrt{5}}{\sqrt{15}} \)
23. The radius of convergence of the series \[ \sum_{n=0}^{\infty} \frac{(n!)^2 x^n}{(2n)!} \] is

(a) \( e^2 \)

(b) \( k \)

(c) 1

(d) 0

(e) 4

24. The series \[ \sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n} \] is

(a) Absolutely convergent

(b) Divergent

(c) Divergent by Comparison Test

(d) Conditionally convergent

(e) Divergent by Ratio Test
25. The value of the integral \( \int_0^1 x^3 \sqrt{1 + x^2} \, dx \) is

(a) \( \frac{2}{15} \sqrt{2} + \frac{2}{15} \)

(b) \( \frac{1}{6} - \frac{\sqrt{2}}{2} \)

(c) \( \sqrt{2} - 1 \)

(d) \( \frac{1}{3} - \sqrt{2} \)

(e) \( \frac{\sqrt{2}}{3} - \frac{1}{15} \)

26. For \( x > 0 \) (but close to 1), the sum of the series \( \sum_{n=0}^{+\infty} \frac{(-2)^n (\ln x)^n}{n!} \) is

(a) \( e^x \)

(b) \( e^{x^2} \)

(c) \( \frac{1}{x^2} \)

(d) \( e^{-2x} \)

(e) \( x^2 \)
27. The value of the integral \( \int_0^{\pi/4} \tan^4 x \sec^4 x \, dx \) is

(a) \( \frac{12}{35} \)

(b) \( \frac{2}{35} \)

(c) \( -\frac{2}{35} \)

(d) \( \frac{5}{12} \)

(e) \( \frac{1}{5} \)

28. The first four terms of the Taylor series of the function \( f(x) = \ln(3 + x) \) at \( a = 1 \) are:

(a) \( \ln 4 - \frac{(x - 1)}{4} + \frac{(x - 1)^2}{32} - \frac{(x - 1)^3}{192} \)

(b) \( \ln 4 + \frac{(x - 1)}{4} - \frac{(x - 1)^2}{32} + \frac{(x - 1)^3}{192} \)

(c) \( \frac{(x - 1)}{4} - \frac{(x - 1)^2}{32} + \frac{(x - 1)^3}{192} - \frac{(x - 1)^4}{256} \)

(d) \( \ln 4 + \frac{(x - 1)}{4} + \frac{(x - 1)^2}{32} + \frac{(x - 1)^3}{192} \)

(e) \( -\frac{(x - 1)}{4} + \frac{(x - 1)^2}{32} - \frac{(x - 1)^3}{192} + \frac{(x - 1)^4}{256} \)
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Final Exam
Second
Saturday, June 11, 2011
Net Time Allowed: 180 minutes

Name: _____________________________________________

ID: ______________________ Sec: ______________________

Check that this exam has 28 questions.

Important Instructions:

1. All types of calculators, pagers or mobile phones are NOT allowed during the examination.

2. Use HB 2.5 pencils only.

3. Use a good eraser. DO NOT use the erasers attached to the pencil.

4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.

5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.

6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.

7. When bubbling, make sure that the bubbled space is fully covered.

8. When erasing a bubble, make sure that you do not leave any trace of penciling.
1. The interval of convergence of the series \[ \sum_{n=1}^{\infty} \frac{(-1)^n(x + 2)^n}{n4^n} \]
is

(a) \((-6, 2)\)
(b) \([-6, 2]\)
(c) \([-2, 2]\)
(d) \([-6, 2]\)
(e) \((-6, 2]\)

2. The value of the integral \[ \int_{0}^{1} \frac{3x + 2}{x^2 - 4} \, dx \]
is

(a) \(3 \ln 2 - \ln 3\)
(b) \(2 \ln 3 - \ln 2\)
(c) \(\ln 3 - 3 \ln 2\)
(d) \(\ln 2 - 2 \ln 3\)
(e) \(\ln 3 - \ln 2\)
3. The series \( \sum_{n=0}^{\infty} \frac{e^{1-2n}}{(\sqrt{2})^{2-2n}} \) is

(a) Convergent and its sum is \( \frac{e^2}{e^2 - 2} \)
(b) Divergent
(c) Convergent and its sum is \( \frac{e}{2} \)
(d) Convergent and its sum is \( \frac{1}{e^3} \)
(e) Convergent and its sum is \( \frac{e^3}{2e^2 - 4} \)

4. The value of the integral \( \int_{-1}^{0} x^2 \sqrt{1+x} \, dx \) is

(a) \( \frac{22}{105} \)
(b) \( \frac{8}{105} \)
(c) \( \frac{19}{105} \)
(d) \( \frac{16}{105} \)
(e) \( \frac{102}{105} \)
5. The series \( \sum_{n=1}^{+\infty} \left( \frac{\cos(n^2)}{n^2} \right)^n \) is

(a) Convergent
(b) Divergent by Ratio Test
(c) Divergent by Divergence Test
(d) Conditionally convergent
(e) Divergent by Comparison Test

6. The sequence \( \left\{ \frac{\ln n}{n^2} \right\}_{n=3} \) is

(a) Neither increasing, nor decreasing and convergent
(b) Increasing and divergent
(c) Increasing and convergent
(d) Decreasing and divergent
(e) Decreasing and convergent
7. The value of the integral \( \int_0^1 x\sqrt{1-x^4} \, dx \) is

(a) \( \frac{\pi}{2} \)

(b) \( \frac{\pi}{3} \)

(c) \( \frac{\pi}{16} \)

(d) \( \frac{\pi}{8} \)

(e) \( \frac{\pi}{6} \)

8. The volume of the solid obtained by rotating the region bounded by the curves \( y = e^{x^2}, y = e \) and \( y = 0 \) (in the first quadrant) about \( y \)-axis is

(a) \( \pi - e^2 \)

(b) \( \pi \)

(c) \( 2\pi \)

(d) \( \pi - e \)

(e) \( e^2\pi \)
9. If \( F(x) = \int_x^{x^2} \frac{dt}{\sqrt{1-t^2}} \), then \( F'(1/2) \) is

(a) \( \frac{4}{\sqrt{15}} - \frac{1}{\sqrt{3}} \)

(b) \( \frac{4 - 2\sqrt{5}}{\sqrt{15}} \)

(c) \( \frac{2 - 2\sqrt{5}}{\sqrt{15}} \)

(d) \( \frac{2 + 2\sqrt{5}}{\sqrt{15}} \)

(e) \( \frac{4 + 2\sqrt{5}}{\sqrt{15}} \)

10. If \( f(x) = f'(x) + 3 \), \( f(0) = 1 \) and \( f(1) = 4 \), then the value of the integral \( \int_0^1 e^x f'(x) \, dx \) is

(a) \( \frac{e + 1}{2} \)

(b) \( \frac{e - 2}{2} \)

(c) \( \frac{2e - 1}{2} \)

(d) \( \frac{e + 2}{2} \)

(e) \( \frac{e - 1}{2} \)
11. The series \( \sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n} \) is

(a) Divergent
(b) Divergent by Comparison Test
(c) Absolutely convergent
(d) Divergent by Ratio Test
(e) Conditionally convergent

12. A power series representing the function \( f(x) = \frac{3x^3}{(x-3)^2} \) is

[Hint: You may consider the function: \( \frac{d}{dx} \left( \frac{1}{3-x} \right) = \frac{1}{(3-x)^2} \)]

(a) \( \sum_{n=1}^{\infty} \frac{x^{n+3}}{3^n} \)
(b) \( \sum_{n=1}^{\infty} \frac{nx^n}{3^{n+2}} \)
(c) \( \sum_{n=1}^{\infty} \frac{x^n}{3^n} \)
(d) \( \sum_{n=1}^{\infty} \frac{nx^n}{3^n} \)
(e) \( \sum_{n=1}^{\infty} \frac{nx^{n+2}}{3^n} \)
13. The area of the region bounded by the curves $4x + y^2 = 12$ and $x = y$ is

(a) $\frac{52}{3}$

(b) $\frac{64}{3}$

(c) 21

(d) $\frac{44}{3}$

(e) 17

14. For which values of $p$, the series $\sum_{n=2}^{\infty} \frac{1}{n \ln n^p}$ is convergent

(a) $p > e$

(b) $p > 1$

(c) converges for all $p$

(d) $p < 1$

(e) diverges for all $p$
15. The length of the curve $y = \ln(\sec x)$, $0 \leq x \leq \frac{\pi}{4}$ is

(a) $\ln(\sqrt{2} + \sqrt{3})$

(b) $\ln(2 + \sqrt{2})$

(c) $\ln(\sqrt{2})$

(d) $1 + \sqrt{2}$

(e) $\ln(1 + \sqrt{2})$

16. The first four terms of the Taylor series of the function $f(x) = \ln(3 + x)$ at $a = 1$ are:

(a) $\ln 4 + \frac{(x - 1)}{4} + \frac{(x - 1)^2}{32} + \frac{(x - 1)^3}{192}$

(b) $\ln 4 - \frac{(x - 1)}{4} + \frac{(x - 1)^2}{32} - \frac{(x - 1)^3}{192}$

(c) $-\frac{(x - 1)}{4} + \frac{(x - 1)^2}{32} - \frac{(x - 1)^3}{192} + \frac{(x - 1)^4}{256}$

(d) $\ln 4 + \frac{(x - 1)}{4} - \frac{(x - 1)^2}{32} + \frac{(x - 1)^3}{192}$

(e) $\frac{(x - 1)}{4} - \frac{(x - 1)^2}{32} + \frac{(x - 1)^3}{192} - \frac{(x - 1)^4}{256}$
17. The value of the integral \[ \int_{0}^{\cos^{-1}(1/e)} (\tan x) \ln(\cos x) \, dx \] is

(a) \( 1/2 \)  
(b) \( -\sqrt{2}/2 \)  
(c) \( -e/2 \)  
(d) \( -1/2 \)  
(e) \( e/2 \)  

18. The value of the integral \[ \int_{0}^{1} x^3 \sqrt{1 + x^2} \, dx \] is

(a) \( \frac{2}{15} \sqrt{2} + \frac{2}{15} \)  
(b) \( \frac{\sqrt{2}}{3} - \frac{1}{15} \)  
(c) \( \frac{1}{3} - \sqrt{2} \)  
(d) \( \frac{1}{6} - \frac{\sqrt{2}}{2} \)  
(e) \( \sqrt{2} - 1 \)
19. The value of the integral \( \int_{0}^{1/3} \frac{x^2 \, dx}{1 + x^7} \) is

(a) \( \sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n + 2) \cdot 3^{7n+2}} \)

(b) \( \sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n + 1) \cdot 3^{7n+2}} \)

(c) \( \sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n + 3) \cdot 3^{7n+3}} \)

(d) \( \sum_{n=0}^{+\infty} \frac{(-1)^n}{7n \cdot (3)^{7n+1}} \)

(e) \( \sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n + 1) \cdot 3^{7n+1}} \)

20. The improper integral \( \int_{1}^{\infty} \frac{e^{-2\sqrt{x}}}{\sqrt{x}} \, dx \) is

(a) Convergent and its value is \( e^{-1} \)

(b) Convergent and its value is \(-e^{-2}\)

(c) Convergent and its value is 0

(d) Convergent and its value is \( e^{-2}\)

(e) Divergent
21. The series \( \sum_{n=0}^{+\infty} \frac{2^n \sin \left( \frac{n\pi}{2} \right)}{n!} \) is

(a) Convergent by Alternating Series Test
(b) Divergent by Divergence Test
(c) Convergent by Limit Comparison Test
(d) Divergent by Ratio Test
(e) Divergent by Comparison Test

22. The radius of convergence of the series \( \sum_{n=0}^{+\infty} \frac{(n!)^2 x^n}{(2n)!} \) is

(a) 4
(b) 1
(c) 0
(d) \( k \)
(e) \( e^2 \)
23. The minimum number of terms needed to estimate the sum of the series \( \sum_{n=1}^{+\infty} \frac{(-1)^n}{(n + 1)^4} \) within 0.0001 (error) is

(a) \( n = 11 \)
(b) \( n = 13 \)
(c) \( n = 10 \)
(d) \( n = 12 \)
(e) \( n = 8 \)

24. The areas of the surface of the solid obtained by rotating the curve \( y = 2\sqrt{x}, \quad 8 \leq x \leq 15 \) about \( x \)-axis is

(a) \( \frac{74\pi}{3} \)
(b) \( \frac{148\pi}{3} \)
(c) \( \frac{96\pi}{3} \)
(d) \( \frac{48\pi}{3} \)
(e) \( \frac{296\pi}{3} \)
25. For $x > 0$ (but close to 1), the sum of the series \[ \sum_{n=0}^{\infty} \frac{(-2)^n \ln x^n}{n!} \] is

(a) $x^2$

(b) $\frac{1}{x^2}$

(c) $e^x$

(d) $e^{x^2}$

(e) $e^{-2x}$

26. The value of the integral \[ \int_0^{\pi/4} \tan^4 x \sec^4 x \, dx \] is

(a) $\frac{1}{5}$

(b) $\frac{5}{12}$

(c) $-\frac{2}{35}$

(d) $\frac{2}{35}$

(e) $\frac{12}{35}$
27. The volume of the solid obtained by rotating the region bounded by the curves $x^2 - y^2 = 1$ and $x = 3$ about the line $x = -2$ is given by

(a) $\int_{-2\sqrt{2}}^{2\sqrt{2}} \pi [9 - (1 + y^2)] dy$

(b) $\int_{0}^{2\sqrt{2}} \pi [25 - (2 + y)^2] dy$

(c) $\int_{-3}^{3} \pi [9 - (x^2 - 1)] dx$

(d) $\int_{-3}^{3} \pi [25 - (2 + \sqrt{x^2 - 1})^2] dx$

(e) $\int_{-2\sqrt{2}}^{2\sqrt{2}} \pi [25 - (2 + \sqrt{1 + y^2})^2] dy$

28. If the sum of the first $n$ terms of a series $\sum_{n=1}^{+\infty} a_n$ is given by $S_n = \frac{2n}{n+1}$, then

(a) $a_n = \frac{2}{n^2 + 2n}$

(b) $a_n = \frac{2}{n^2 + 1}$

(c) $a_n = \frac{4}{n^2 + n}$

(d) $a_n = \frac{1}{n^2 + n}$

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King Fahd University of Petroleum & Minerals  
Department of Mathematics and Statistics

CODE 003 Math 102 CODE 003

Final Exam  
Second  
Saturday, June 11, 2011  
Net Time Allowed: 180 minutes

Name: ___________________________________

ID: ____________________ Sec: ________________

Check that this exam has 28 questions.

Important Instructions:

1. All types of calculators, pagers or mobile phones are NOT allowed during the examination.

2. Use HB 2.5 pencils only.

3. Use a good eraser. DO NOT use the erasers attached to the pencil.

4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.

5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.

6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.

7. When bubbling, make sure that the bubbled space is fully covered.

8. When erasing a bubble, make sure that you do not leave any trace of penciling.
1. The value of the integral $\int_0^{1/3} \frac{x^2 dx}{1 + x^3}$ is

(a) $\sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n + 3) \cdot 3^{7n+3}}$

(b) $\sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n + 1) \cdot 3^{7n+1}}$

(c) $\sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n + 1) \cdot 3^{7n+2}}$

(d) $\sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n + 2) \cdot 3^{7n+2}}$

(e) $\sum_{n=0}^{+\infty} \frac{(-1)^n}{7n \cdot (3)^{7n+1}}$

2. The length of the curve $y = \ln(\sec x)$, $0 \leq x \leq \frac{\pi}{4}$ is

(a) $\ln(\sqrt{2} + \sqrt{3})$

(b) $\ln(\sqrt{2})$

(c) $\ln(1 + \sqrt{2})$

(d) $1 + \sqrt{2}$

(e) $\ln(2 + \sqrt{2})$
3. The first four terms of the Taylor series of the function \( f(x) = \ln(3 + x) \) at \( a = 1 \) are:

(a) \( \ln 4 + \frac{(x - 1)}{4} + \frac{(x - 1)^2}{32} + \frac{(x - 1)^3}{192} \)

(b) \( \ln 4 - \frac{(x - 1)}{4} + \frac{(x - 1)^2}{32} - \frac{(x - 1)^3}{192} \)

(c) \( \ln 4 + \frac{(x - 1)}{4} - \frac{(x - 1)^2}{32} + \frac{(x - 1)^3}{192} \)

(d) \( \frac{(x - 1)}{4} - \frac{(x - 1)^2}{32} + \frac{(x - 1)^3}{192} - \frac{(x - 1)^4}{256} \)

(e) \( -\frac{(x - 1)}{4} + \frac{(x - 1)^2}{32} - \frac{(x - 1)^3}{192} + \frac{(x - 1)^4}{256} \)

4. The value of the integral \( \int_{-1}^{0} x^2 \sqrt{1 + x} \, dx \) is

(a) \( \frac{19}{105} \)

(b) \( \frac{102}{105} \)

(c) \( \frac{22}{105} \)

(d) \( \frac{16}{105} \)

(e) \( \frac{8}{105} \)
5. For which values of \( p \), the series \( \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p} \) is convergent

(a) \( p > 1 \)
(b) \( p < 1 \)
(c) \( p > e \)
(d) converges for all \( p \)
(e) diverges for all \( p \)

6. The minimum number of terms needed to estimate the sum of the series \( \sum_{n=1}^{+\infty} \frac{(-1)^n}{(n + 1)^4} \) within 0.0001 (error) is

(a) \( n = 10 \)
(b) \( n = 8 \)
(c) \( n = 13 \)
(d) \( n = 12 \)
(e) \( n = 11 \)
7. The radius of convergence of the series \( \sum_{n=0}^{+\infty} \frac{(n!)^2 x^n}{(2n)!} \) is

(a) 1
(b) 0
(c) 4
(d) \(e^2\)
(e) \(k\)

8. The area of the region bounded by the curves \(4x + y^2 = 12\) and \(x = y\) is

(a) \(\frac{52}{3}\)
(b) \(\frac{44}{3}\)
(c) 17
(d) \(\frac{64}{3}\)
(e) 21
9. A power series representing the function \( f(x) = \frac{3x^3}{(x - 3)^2} \) is

[Hint: You may consider the function: \( \frac{d}{dx} \left( \frac{1}{3 - x} \right) = \frac{1}{(3 - x)^2} \)]

(a) \( \sum_{n=1}^{+\infty} \frac{nx^n}{3^{n+2}} \)

(b) \( \sum_{n=1}^{+\infty} \frac{x^{n+3}}{3^n} \)

(c) \( \sum_{n=1}^{+\infty} \frac{nx^{n+2}}{3^n} \)

(d) \( \sum_{n=1}^{+\infty} \frac{nx^n}{3^n} \)

(e) \( \sum_{n=1}^{+\infty} \frac{x^n}{3^n} \)

10. The sequence \( \left\{ \frac{\ln n}{n^2} \right\}_{n \geq 3} \) is

(a) Decreasing and convergent

(b) Increasing and divergent

(c) Decreasing and divergent

(d) Increasing and convergent

(e) Neither increasing, nor decreasing and convergent
11. The value of the integral \( \int_0^1 \frac{3x + 2}{x^2 - 4} \, dx \) is

(a) \( \ln 3 - \ln 2 \)
(b) \( \ln 3 - 3 \ln 2 \)
(c) \( \ln 2 - 2 \ln 3 \)
(d) \( 3 \ln 2 - \ln 3 \)
(e) \( 2 \ln 3 - \ln 2 \)

12. The value of the integral \( \int_0^1 x^3 \sqrt{1 + x^2} \, dx \) is

(a) \( \sqrt{2} - 1 \)
(b) \( \frac{\sqrt{2}}{3} - \frac{1}{15} \)
(c) \( \frac{1}{3} - \sqrt{2} \)
(d) \( \frac{2}{15} \sqrt{2} + \frac{2}{15} \)
(e) \( \frac{1}{6} - \frac{\sqrt{2}}{2} \)
13. If \( f(x) = f'(x) + 3 \), \( f(0) = 1 \) and \( f(1) = 4 \), then the value of the integral \( \int_0^1 e^xf'(x)dx \) is

(a) \( \frac{e + 1}{2} \)

(b) \( \frac{e - 2}{2} \)

(c) \( \frac{2e - 1}{2} \)

(d) \( \frac{e - 1}{2} \)

(e) \( \frac{e + 2}{2} \)

14. The areas of the surface of the solid obtained by rotating the curve \( y = 2\sqrt{x} \), \( 8 \leq x \leq 15 \) about \( x \)-axis is

(a) \( \frac{296\pi}{3} \)

(b) \( \frac{48\pi}{3} \)

(c) \( \frac{148\pi}{3} \)

(d) \( \frac{96\pi}{3} \)

(e) \( \frac{74\pi}{3} \)
15. The interval of convergence of the series \( \sum_{n=1}^{\infty} \frac{(-1)^n(x + 2)^n}{n^4} \) is

(a) \([-6, 2]\)
(b) \([-2, 2]\)
(c) \((-6, 2)\)
(d) \((-6, 2]\)
(e) \([-6, 2)\)

16. The value of the integral \( \int_{0}^{\cos^{-1}(1/e)} (\tan x) \ln(\cos x) dx \) is

(a) \(e/2\)
(b) \(-1/2\)
(c) \(1/2\)
(d) \(-\sqrt{2}/2\)
(e) \(-e/2\)
17. The series \( \sum_{n=0}^{\infty} \frac{e^{1-2n}}{(\sqrt{2})^{2-2n}} \) is

(a) Divergent

(b) Convergent and its sum is \( \frac{e}{2} \)

(c) Convergent and its sum is \( \frac{e^2}{e^2 - 2} \)

(d) Convergent and its sum is \( \frac{e^3}{2e^2 - 4} \)

(e) Convergent and its sum is \( \frac{1}{e^3} \)

18. The volume of the solid obtained by rotating the region bounded by the curves \( y = e^{x^2}, y = e \) and \( y = 0 \) (in the first quadrant) about \( y \)-axis is

(a) \( e^2 \pi \)

(b) \( \pi - e \)

(c) \( \pi - e^2 \)

(d) \( \pi \)

(e) \( 2\pi \)
19. The series \( \sum_{n=1}^{+\infty} \left( \frac{\cos(n^2)}{n^2} \right)^n \) is

(a) Conditionally convergent
(b) Divergent by Divergence Test
(c) Divergent by Comparison Test
(d) Divergent by Ratio Test
(e) Convergent

20. If the sum of the first \( n \) terms of a series \( \sum_{n=1}^{+\infty} a_n \) is given by
   \( S_n = \frac{2n}{n+1} \), then

(a) \( a_n = \frac{2}{n^2 + 2n} \)
(b) \( a_n = \frac{1}{n^2 + n} \)
(c) \( a_n = \frac{2}{n^2 + n} \)
(d) \( a_n = \frac{2}{n^2 + 1} \)
(e) \( a_n = \frac{4}{n^2 + n} \)
21. The value of the integral \( \int_0^1 x \sqrt{1 - x^4} \, dx \) is

(a) \( \frac{\pi}{16} \)

(b) \( \frac{\pi}{2} \)

(c) \( \frac{\pi}{3} \)

(d) \( \frac{\pi}{6} \)

(e) \( \frac{\pi}{8} \)

22. The improper integral \( \int_1^\infty \frac{e^{-2\sqrt{x}}}{\sqrt{x}} \, dx \) is

(a) Convergent and its value is \( e^{+2} \)

(b) Divergent

(c) Convergent and its value is \( e^{-2} \)

(d) Convergent and its value is \( -e^{-2} \)

(e) Convergent and its value is 0
23. For \( x > 0 \) (but close to 1), the sum of the series \( \sum_{n=0}^{\infty} \frac{(-2)^n (\ln x)^n}{n!} \) is

(a) \( e^{x^2} \)
(b) \( e^x \)
(c) \( \frac{1}{x^2} \)
(d) \( e^{-2x} \)
(e) \( x^2 \)

24. If \( F(x) = \int_{x}^{x^2} \frac{dt}{\sqrt{1-t^2}} \), then \( F'(\frac{1}{2}) \) is

(a) \( \frac{2 + 2\sqrt{5}}{\sqrt{15}} \)
(b) \( \frac{4}{\sqrt{15}} - \frac{1}{\sqrt{3}} \)
(c) \( \frac{4 - 2\sqrt{5}}{\sqrt{15}} \)
(d) \( \frac{2 - 2\sqrt{5}}{\sqrt{15}} \)
(e) \( \frac{4 + 2\sqrt{5}}{\sqrt{15}} \)
25. The volume of the solid obtained by rotating the region bounded by the curves $x^2 - y^2 = 1$ and $x = 3$ about the line $x = -2$ is given by

(a) \[ \int_{0}^{2\sqrt{2}} \pi [25 - (2 + y)^2] \, dy \]

(b) \[ \int_{-3}^{3} \pi [25 - (2 + \sqrt{x^2 - 1})^2] \, dx \]

(c) \[ \int_{-3}^{3} \pi [9 - (x^2 - 1)] \, dx \]

(d) \[ \int_{-2\sqrt{2}}^{2\sqrt{2}} \pi [25 - \left(2 + \sqrt{1 + y^2}\right)^2] \, dy \]

(e) \[ \int_{-2\sqrt{2}}^{2\sqrt{2}} \pi [9 - (1 + y^2)] \, dy \]

26. The series \[ \sum_{n=1}^{+\infty} \frac{\cos(n\pi)}{n} \] is

(a) Divergent by Ratio Test

(b) Divergent

(c) Divergent by Comparison Test

(d) Conditionally convergent

(e) Absolutely convergent
27. The value of the integral $\int_0^{\pi/4} \tan^4 x \sec^4 x \, dx$ is

(a) $\frac{5}{12}$

(b) $\frac{12}{35}$

(c) $\frac{1}{5}$

(d) $\frac{2}{35}$

(e) $-\frac{2}{35}$

28. The series $\sum_{n=0}^{+\infty} \frac{2^n \sin \left( n \frac{\pi}{2} \right)}{n!}$ is

(a) Divergent by Ratio Test

(b) Convergent by Alternating Series Test

(c) Convergent by Limit Comparison Test

(d) Divergent by Comparison Test

(e) Divergent by Divergence Test
|   | a   | b   | c   | d   | e   | f   |   | a   | b   | c   | d   | e   | f   |   | a   | b   | c   | d   | e   | f   |   | a   | b   | c   | d   | e   | f   |   | a   | b   | c   | d   | e   | f   |   | a   | b   | c   | d   | e   | f   |   | a   | b   | c   | d   | e   | f   |
|---|-----|-----|-----|-----|-----|-----|---|-----|-----|-----|-----|-----|-----|---|-----|-----|-----|-----|-----|---|-----|-----|-----|-----|---|-----|-----|-----|-----|-----|---|-----|-----|-----|-----|-----|---|-----|-----|-----|-----|-----|---|-----|-----|-----|-----|-----|---|-----|-----|-----|-----|-----|---|-----|-----|-----|-----|-----|---|-----|-----|-----|-----|-----|
| 1 | a   | b   | c   | d   | e   | f   | 2 | a   | b   | c   | d   | e   | f   | 3 | a   | b   | c   | d   | e   | f   | 4 | a   | b   | c   | d   | e   | f   | 5 | a   | b   | c   | d   | e   | f   | 6 | a   | b   | c   | d   | e   | f   | 7 | a   | b   | c   | d   | e   | f   | 8 | a   | b   | c   | d   | e   | f   | 9 | a   | b   | c   | d   | e   | f   | 10| a   | b   | c   | d   | e   | f   | 11| a   | b   | c   | d   | e   | f   | 12| a   | b   | c   | d   | e   | f   | 13| a   | b   | c   | d   | e   | f   | 14| a   | b   | c   | d   | e   | f   | 15| a   | b   | c   | d   | e   | f   | 16| a   | b   | c   | d   | e   | f   | 17| a   | b   | c   | d   | e   | f   | 18| a   | b   | c   | d   | e   | f   | 19| a   | b   | c   | d   | e   | f   | 20| a   | b   | c   | d   | e   | f   | 21| a   | b   | c   | d   | e   | f   | 22| a   | b   | c   | d   | e   | f   | 23| a   | b   | c   | d   | e   | f   | 24| a   | b   | c   | d   | e   | f   | 25| a   | b   | c   | d   | e   | f   | 26| a   | b   | c   | d   | e   | f   | 27| a   | b   | c   | d   | e   | f   | 28| a   | b   | c   | d   | e   | f   | 29| a   | b   | c   | d   | e   | f   | 30| a   | b   | c   | d   | e   | f   | 31| a   | b   | c   | d   | e   | f   | 32| a   | b   | c   | d   | e   | f   | 33| a   | b   | c   | d   | e   | f   | 34| a   | b   | c   | d   | e   | f   | 35| a   | b   | c   | d   | e   | f   | 36| a   | b   | c   | d   | e   | f   | 37| a   | b   | c   | d   | e   | f   | 38| a   | b   | c   | d   | e   | f   | 39| a   | b   | c   | d   | e   | f   | 40| a   | b   | c   | d   | e   | f   | 41| a   | b   | c   | d   | e   | f   | 42| a   | b   | c   | d   | e   | f   | 43| a   | b   | c   | d   | e   | f   | 44| a   | b   | c   | d   | e   | f   | 45| a   | b   | c   | d   | e   | f   | 46| a   | b   | c   | d   | e   | f   | 47| a   | b   | c   | d   | e   | f   | 48| a   | b   | c   | d   | e   | f   | 49| a   | b   | c   | d   | e   | f   | 50| a   | b   | c   | d   | e   | f   | 51| a   | b   | c   | d   | e   | f   | 52| a   | b   | c   | d   | e   | f   | 53| a   | b   | c   | d   | e   | f   | 54| a   | b   | c   | d   | e   | f   | 55| a   | b   | c   | d   | e   | f   | 56| a   | b   | c   | d   | e   | f   | 57| a   | b   | c   | d   | e   | f   | 58| a   | b   | c   | d   | e   | f   | 59| a   | b   | c   | d   | e   | f   | 60| a   | b   | c   | d   | e   | f   | 61| a   | b   | c   | d   | e   | f   | 62| a   | b   | c   | d   | e   | f   | 63| a   | b   | c   | d   | e   | f   | 64| a   | b   | c   | d   | e   | f   | 65| a   | b   | c   | d   | e   | f   | 66| a   | b   | c   | d   | e   | f   | 67| a   | b   | c   | d   | e   | f   | 68| a   | b   | c   | d   | e   | f   | 69| a   | b   | c   | d   | e   | f   | 70| a   | b   | c   | d   | e   | f   |
Name: 

ID: ______________ Sec: ______________

Check that this exam has 28 questions.

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7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.
1. The volume of the solid obtained by rotating the region bounded by the curves \( x^2 - y^2 = 1 \) and \( x = 3 \) about the line \( x = -2 \) is given by

(a) \( \int_{-3}^{3} \pi [25 - (2 + \sqrt{x^2 - 1})^2] \, dx \)

(b) \( \int_{-3}^{3} \pi [9 - (x^2 - 1)] \, dx \)

(c) \( \int_{0}^{2\sqrt{2}} \pi [25 - (2 + y)^2] \, dy \)

(d) \( \int_{-2\sqrt{2}}^{2\sqrt{2}} \pi [25 - (2 + \sqrt{1 + y^2})^2] \, dy \)

(e) \( \int_{-2\sqrt{2}}^{2\sqrt{2}} \pi [9 - (1 + y^2)] \, dy \)

2. The value of the integral \( \int_{0}^{1} x^3 \sqrt{1 + x^2} \, dx \) is

(a) \( \frac{\sqrt{2}}{3} - \frac{1}{15} \)

(b) \( \sqrt{2} - 1 \)

(c) \( \frac{1}{6} - \frac{\sqrt{2}}{2} \)

(d) \( \frac{1}{3} - \sqrt{2} \)

(e) \( \frac{2}{15} \sqrt{2} + \frac{2}{15} \)
3. The areas of the surface of the solid obtained by rotating the curve \( y = 2\sqrt{x}, \quad 8 \leq x \leq 15 \) about \( x \)-axis is

(a) \( \frac{48\pi}{3} \)

(b) \( \frac{296\pi}{3} \)

(c) \( \frac{74\pi}{3} \)

(d) \( \frac{148\pi}{3} \)

(e) \( \frac{96\pi}{3} \)

4. The radius of convergence of the series \( \sum_{n=0}^{\infty} \frac{(n!)^2 x^n}{(2n)!} \) is

(a) \( e^2 \)

(b) 4

(c) \( k \)

(d) 0

(e) 1
5. The volume of the solid obtained by rotating the region bounded by the curves \( y = e^{x^2}, \ y = e \) and \( y = 0 \) (in the first quadrant) about \( y \)-axis is

(a) \( \pi - e \)

(b) \( 2\pi \)

(c) \( \pi \)

(d) \( \pi - e^2 \)

(e) \( e^2 \pi \)

6. For which values of \( p \), the series \( \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p} \) is convergent

(a) \( p > e \)

(b) diverges for all \( p \)

(c) \( p < 1 \)

(d) converges for all \( p \)

(e) \( p > 1 \)
7. For $x > 0$ (but close to 1), the sum of the series $\sum_{n=0}^{\infty} \frac{(-2)^n(\ln x)^n}{n!}$ is

(a) $x^2$

(b) $\frac{1}{x^2}$

(c) $e^{x^2}$

(d) $e^x$

(e) $e^{-2x}$

8. The sequence $\left\{\frac{\ln n}{n^2}\right\}_{n\geq3}$ is

(a) Increasing and divergent

(b) Decreasing and divergent

(c) Decreasing and convergent

(d) Increasing and convergent

(e) Neither increasing, nor decreasing and convergent
9. The series \( \sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n} \) is

(a) Divergent by Ratio Test

(b) Divergent by Comparison Test

(c) Divergent

(d) Conditionally convergent

(e) Absolutely convergent

10. The series \( \sum_{n=0}^{\infty} \frac{e^{1-2n}}{(\sqrt{2})^{2-2n}} \) is

(a) Divergent

(b) Convergent and its sum is \( \frac{e^3}{2e^2 - 4} \)

(c) Convergent and its sum is \( \frac{1}{e^3} \)

(d) Convergent and its sum is \( \frac{e}{2} \)

(e) Convergent and its sum is \( \frac{e^2}{e^2 - 2} \)
11. The length of the curve \( y = \ln(\sec x), \; 0 \leq x \leq \frac{\pi}{4} \) is

(a) \( 1 + \sqrt{2} \)

(b) \( \ln(\sqrt{2}) \)

(c) \( \ln(2 + \sqrt{2}) \)

(d) \( \ln(1 + \sqrt{2}) \)

(e) \( \ln(\sqrt{2} + \sqrt{3}) \)

12. The value of the integral \( \int_{-1}^{0} x^2 \sqrt{1 + x} \, dx \) is

(a) \( \frac{16}{105} \)

(b) \( \frac{102}{105} \)

(c) \( \frac{19}{105} \)

(d) \( \frac{22}{105} \)

(e) \( \frac{8}{105} \)
13. The minimum number of terms needed to estimate the sum of the series \[ \sum_{n=1}^{\infty} \frac{(-1)^n}{(n+1)^4} \] within 0.0001 (error) is

   (a) \( n = 8 \)
   
   (b) \( n = 10 \)
   
   (c) \( n = 11 \)
   
   (d) \( n = 13 \)
   
   (e) \( n = 12 \)

14. The value of the integral \( \int_{0}^{1} \frac{3x + 2}{x^2 - 4} \, dx \) is

   (a) \( 3 \ln 2 - \ln 3 \)
   
   (b) \( \ln 3 - 3 \ln 2 \)
   
   (c) \( \ln 3 - \ln 2 \)
   
   (d) \( 2 \ln 3 - \ln 2 \)
   
   (e) \( \ln 2 - 2 \ln 3 \)
15. The interval of convergence of the series \( \sum_{n=1}^{\infty} \frac{(-1)^n(x + 2)^n}{n4^n} \) is

(a) \([-6, 2]\)

(b) \((-6, 2]\)

(c) \([-2, 2]\)

(d) \([-6, 2)\)

(e) \((-6, 2)\)

16. If \( F(x) = \int_{x}^{x^2} \frac{dt}{\sqrt{1-t^2}} \), then \( F'(\frac{1}{2}) \) is

(a) \( \frac{4}{\sqrt{15}} - \frac{1}{\sqrt{3}} \)

(b) \( \frac{4 - 2\sqrt{5}}{\sqrt{15}} \)

(c) \( \frac{4 + 2\sqrt{5}}{\sqrt{15}} \)

(d) \( \frac{2 - 2\sqrt{5}}{\sqrt{15}} \)

(e) \( \frac{2 + 2\sqrt{5}}{\sqrt{15}} \)
17. The series \( \sum_{n=0}^{+\infty} \frac{2^n \sin \left( \frac{n\pi}{2} \right)}{n!} \) is

(a) Convergent by Limit Comparison Test
(b) Convergent by Alternating Series Test
(c) Divergent by Divergence Test
(d) Divergent by Comparison Test
(e) Divergent by Ratio Test

18. If the sum of the first \( n \) terms of a series \( \sum_{n=1}^{+\infty} a_n \) is given by \( S_n = \frac{2n}{n+1} \), then

(a) \( a_n = \frac{2}{n^2 + n} \)
(b) \( a_n = \frac{2}{n^2 + 1} \)
(c) \( a_n = \frac{2}{n^2 + 2n} \)
(d) \( a_n = \frac{1}{n^2 + n} \)
(e) \( a_n = \frac{4}{n^2 + n} \)
19. The value of the integral \( \int_0^1 x \sqrt{1 - x^4} \, dx \) is

(a) \( \frac{\pi}{8} \)

(b) \( \frac{\pi}{16} \)

(c) \( \frac{\pi}{3} \)

(d) \( \frac{\pi}{6} \)

(e) \( \frac{\pi}{2} \)

20. The improper integral \( \int_1^\infty \frac{e^{-2\sqrt{x}}}{\sqrt{x}} \, dx \) is

(a) Convergent and its value is \( e^{-2} \)

(b) Convergent and its value is \( -e^{-2} \)

(c) Divergent

(d) Convergent and its value is 0

(e) Convergent and its value is \( e^{-2} \)
21. The series \( \sum_{n=1}^{\infty} \left( \frac{\cos(n^2)}{n^2} \right)^n \) is

(a) Conditionally convergent

(b) Divergent by Divergence Test

(c) Convergent

(d) Divergent by Ratio Test

(e) Divergent by Comparison Test

22. The first four terms of the Taylor series of the function \( f(x) = \ln(3 + x) \) at \( a = 1 \) are:

(a) \( \ln 4 + \frac{(x - 1)}{4} - \frac{(x - 1)^2}{32} + \frac{(x - 1)^3}{192} \)

(b) \( \frac{(x - 1)}{4} - \frac{(x - 1)^2}{32} + \frac{(x - 1)^3}{192} - \frac{(x - 1)^4}{256} \)

(c) \( -\frac{(x - 1)}{4} + \frac{(x - 1)^2}{32} - \frac{(x - 1)^3}{192} + \frac{(x - 1)^4}{256} \)

(d) \( \ln 4 + \frac{(x - 1)}{4} + \frac{(x - 1)^2}{32} + \frac{(x - 1)^3}{192} \)

(e) \( \ln 4 - \frac{(x - 1)}{4} + \frac{(x - 1)^2}{32} - \frac{(x - 1)^3}{192} \)
23. The value of the integral \( \int_{0}^{\pi/4} \tan^4 x \sec^4 x \, dx \) is

(a) \( \frac{2}{35} \)

(b) \( \frac{12}{35} \)

(c) \( \frac{1}{5} \)

(d) \( \frac{5}{12} \)

(e) \( -\frac{2}{35} \)

24. The value of the integral \( \int_{0}^{1/3} \frac{x^2 \, dx}{1 + x^7} \) is

(a) \( \sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n + 3) \cdot 3^{7n+3}} \)

(b) \( \sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n + 2) \cdot 3^{7n+2}} \)

(c) \( \sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n + 1) \cdot 3^{7n+2}} \)

(d) \( \sum_{n=0}^{+\infty} \frac{(-1)^n}{7n \cdot (3)^{7n+1}} \)

(e) \( \sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n + 1) \cdot 3^{7n+1}} \)
25. A power series representing the function \( f(x) = \frac{3x^3}{(x - 3)^2} \) is

\[ \text{[Hint: You may consider the function: } \frac{d}{dx} \left( \frac{1}{3 - x} \right) = \frac{1}{(3 - x)^2} \] 

\[ + \infty \sum_{n=1}^{\infty} \frac{nx^n}{3^n} \]

(a) \[ + \infty \sum_{n=1}^{\infty} \frac{nx^{n+2}}{3^n} \]

(b) \[ + \infty \sum_{n=1}^{\infty} \frac{nx^n}{3^n} \]

(c) \[ + \infty \sum_{n=1}^{\infty} \frac{x^{n+3}}{3^n} \]

(d) \[ + \infty \sum_{n=1}^{\infty} \frac{x^n}{3^n} \]

(e) \[ + \infty \sum_{n=1}^{\infty} \frac{nx^n}{3^{n+2}} \]

26. If \( f(x) = f'(x) + 3 \), \( f(0) = 1 \) and \( f(1) = 4 \), then the value of the integral \( \int_0^1 e^x f'(x) dx \) is

(a) \[ \frac{e + 1}{2} \]

(b) \[ \frac{e - 2}{2} \]

(c) \[ \frac{e - 1}{2} \]

(d) \[ \frac{2e - 1}{2} \]

(e) \[ \frac{e + 2}{2} \]
27. The value of the integral \( \int_{0}^{\cos^{-1}(1/e)} (\tan x) \ln(\cos x) \, dx \) is

(a) \( 1/2 \)

(b) \( -1/2 \)

(c) \( e/2 \)

(d) \( -e/2 \)

(e) \( -\sqrt{2}/2 \)

28. The area of the region bounded by the curves \( 4x + y^2 = 12 \) and \( x = y \) is

(a) \( \frac{64}{3} \)

(b) 17

(c) \( \frac{44}{3} \)

(d) 21

(e) \( \frac{52}{3} \)
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