

Problem 1

Solve the integral $\int_e^{e^2} \frac{dx}{x \ln x}$.

Set $u = \ln x$, $du = \frac{dx}{x}$ and $x = e \Rightarrow u = 1$, $x = e^2 \Rightarrow u = 2$. Then

$$\int_e^{e^2} \frac{dx}{x \ln x} = \int_1^2 \frac{du}{u} = [\ln u]_1^2 = \ln 2$$

Problem 2

Compute the area bounded by the curves $y = x^3 - x$ and $y = x^2 - 1$.

To find the points of intersection, we let $x^3 - x = x^2 - 1 \Leftrightarrow x(x^2 - 1) = x^2 - 1 \Leftrightarrow (x - 1)(x^2 - 1) = 0 \Leftrightarrow (x - 1)^2(x + 1) = 0 \Rightarrow x = 1$ or $x = -1$. Within the interval $[-1, 1]$, the curve $y = x^3 - x$ sits on top of $y = x^2 - 1$. Hence the area is

$$\int_{-1}^1 (x^3 - x - x^2 + 1) dx = \left[\frac{x^4}{4} - \frac{x^2}{2} - \frac{x^3}{3} + x \right]_{-1}^1 = \frac{1}{4} - \frac{1}{2} - \frac{1}{3} + 1 - \frac{1}{4} + \frac{1}{2} - \frac{1}{3} + 1 = 2 - \frac{2}{3} = \frac{4}{3}$$