Student ID: Name: Section:

Problem 1

Solve the integral $\int \frac{dx}{x^4 - x^2}$. Show all your work clearly.

$$\frac{1}{x^4 - x^2} = \frac{1}{x^2(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} + \frac{D}{x+1} = \frac{(A+C+D)x^3 + (B+C-D)x^2 - Ax - B}{x^4 - x^2}$$

Therefore $B = -1$, $A = 0$, $C + D = 0$, $C - D = 1$ which implies $C = \frac{1}{2}$, $D = -\frac{1}{2}$.

Then $\int \frac{dx}{x^4 - x^2} = -\int \frac{dx}{x^2} + \frac{1}{2}\int \frac{dx}{x-1} - \frac{1}{2}\int \frac{dx}{x+1} = \frac{1}{x} + \ln\left(\sqrt{\frac{x-1}{x+1}}\right) + c$

Problem 2

Solve the integral $\displaystyle\int_0^1 \frac{e^{1/x}}{x^3}\,dx$. Show all your work clearly.

$\displaystyle\int_0^1 \frac{e^{1/x}}{x^3}\,dx = \lim_{t\to 0^+}\left(\int_t^1 \frac{e^{1/x}}{x^3}\,dx\right)$. Let $u = \frac{1}{x}$, $du = -\frac{1}{x^2}$, then $\displaystyle\int_t^1 \frac{e^{1/x}}{x^3}\,dx = -\int_{1/t}^1 ue^u\,du = -[ue^u]_{1/t}^1 + \int_{1/t}^1 e^u\,du = -e + \frac{e^{1/t}}{t} + e - e^{1/t}$. Hence we get the limit $\displaystyle\lim_{t\to 0^+}\left(\frac{e^{1/t}}{t} - e^{1/t}\right)$ which is equal to $\displaystyle\lim_{t\to 0^+}\left(\frac{1-t}{t}e^{1/t}\right)$. Both terms $\dfrac{1-t}{t}$ and $e^{1/t}$ go to $+\infty$, so the integral is divergent.