

$$b) a_n = \frac{1-2\cos(n)}{\ln(n)}$$

$$-1 \leq \cos(n) \leq 1 \Rightarrow -2 \leq -2\cos(n) \leq 2$$

↓

$$-1 \leq 1-2\cos(n) \leq 3$$

↓

$$-\frac{1}{\ln(n)} \leq \frac{1-2\cos(n)}{\ln(n)} \leq \frac{3}{\ln(n)}$$

$$\downarrow_{n \rightarrow \infty} \\ 0$$

$$\downarrow_{n \rightarrow \infty} \\ 0$$

$0/0$, $\lim_{n \rightarrow \infty} \frac{1-2\cos(n)}{\ln(n)} = 0$ (by squeeze theorem)

and hence, $\left\{ \frac{1-2\cos(n)}{\ln(n)} \right\}_{n=2}^{\infty}$ is convergent.

Q3) Find the sum of the series: $\sum_{n=0}^{\infty} \frac{1}{n^2+3n+2}$

$$n^2 + 3n + 2 = (n+1)(n+2), \text{ so,}$$

$$\sum_{n=0}^{\infty} \frac{1}{(n^2+3n+2)} = \sum_{n=0}^{\infty} \frac{1}{(n+1)(n+2)} = \sum_{n=0}^{\infty} \left[\frac{1}{n+1} - \frac{1}{n+2} \right]$$

$$S_n = \sum_{i=1}^n a_i = \sum_{i=1}^n \left(1 - \frac{1}{n+1} \right)$$

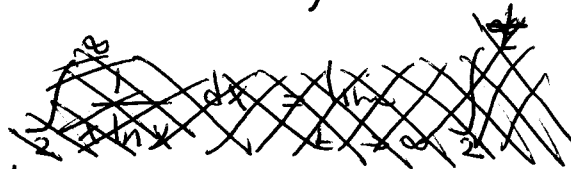
$$= \sum_{n=1}^{\infty} \left[\frac{1}{n} - \frac{1}{n+1} \right] = a_n$$

$$\lim_{n \rightarrow \infty} S_n = 1 \quad \circ \circ \quad \sum_{n=0}^{\infty} \frac{1}{(n^2+3n+2)} = \lim_{n \rightarrow \infty} S_n = 1$$

Q4) Test the series $\sum_{n=2}^{\infty} \frac{1}{n \ln(n)}$ for convergence or divergence.

$a_n = \frac{1}{n \ln(n)}$ $\therefore a_n$ is continuous, positive & decreasing on $[2, \infty)$.

Let $f(x) = \frac{1}{x \ln x}$



$$\int_2^t \frac{1}{x \ln x} dx = \int_{\ln 2}^{\ln t} \frac{du}{u} = \ln(\ln t) - \ln(\ln 2) \rightarrow \infty \text{ as } t \rightarrow \infty$$

$u \Rightarrow du = \frac{dx}{x}$

$\therefore \int_2^{\infty} \frac{1}{x \ln x} dx$ is divergent and then $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ is divergent by the integral test.