

Ex1: Find $\frac{dy}{dx}$ for $y = \sqrt{\frac{x-2}{x+3}}$

Solution:

$$y = \left(\frac{x-2}{x+3}\right)^{\frac{1}{2}}, \text{ So } \frac{dy}{dx} = \frac{1}{2} \cdot \frac{d}{dx} \left(\frac{x-2}{x+3}\right) \cdot \left(\frac{x-2}{x+3}\right)^{\frac{1}{2}-1}$$

Thus:

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{2} \frac{x+3 - x+2}{(x+3)^2} \left(\frac{x-2}{x+3}\right)^{-\frac{1}{2}} = \frac{5}{2(x+3)^2} \left(\frac{x+3}{x-2}\right)^{\frac{1}{2}} \\ &= \frac{5}{2(x+3)^{\frac{3}{2}}(x-2)^{\frac{1}{2}}} \end{aligned}$$

Ex2: If $z = 2y^2 - 4y + 5$, $y = 6x - 5$ and $x = 2t$, use the Chain rule to find $\frac{dz}{dt} \Big|_{t=1}$.

Solution: By the chain rule we have that:

$$\frac{dz}{dt} = \frac{dz}{dy} \frac{dy}{dx} \frac{dx}{dt} = (4y-4) \cdot 6 \cdot 2 = 48(y-1)$$

Now, when $t = 1$, $x = 2$. Therefore $y = 12 - 5 = 7$.

$$\text{Hence } \frac{dz}{dt} \Big|_{t=1} = 48(7-1) = 288$$

Ex3: Find an equation of the tangent line to $y = \frac{-3}{(3x^2+1)^3}$ at $x=0$.

Solution: $y = -3(3x^2+1)^{-3}$

$$\frac{dy}{dx} = -3 \left[-3(6x)(3x^2+1)^{-4} \right] = \frac{54x}{(3x^2+1)^4}. \text{ The slope is given}$$

by $\frac{dy}{dx} \Big|_{x=0} = 0$. Now $y(0) = -3$. Hence an equation of

the tangent line is: $y = -3$.