Math 132 Quiz 2 Date 03/02/2011

Ex1: Consider $f(x) = \begin{cases} 0 & \text{if } x \leq 1 \\ x-1 & \text{if } x > 1 \end{cases}$

a) Find $\lim\limits_{x \to 1} f(x)$

b) Is f continuous at 1? Justify your answer.

Ex2: Let $f(x) = \dfrac{3}{x-1}$

a) Use the definition of the derivative to find the slope of the tangent line to the curve $y = f(x)$ at $x = 2$

b) Give an equation of the tangent line to the curve $y = f(x)$ at $x = 2$.

Solution

Ex1 a) $\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+} x - 1 = 1 - 1 = 0$

$\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-} 0 = 0$

Since $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^+} f(x) = 0$, the limit $\lim\limits_{x \to 1} f(x)$ exists and $\lim\limits_{x \to 1} f(x) = 0$.

b) $f(1) = 0 = \lim\limits_{x \to 1} f(x)$. Hence f is continuous at 1.

Ex2:

a) $$\lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \to 0} \frac{\frac{3}{2+h-1} - 3}{h} = \lim_{h \to 0} \frac{\frac{3}{1+h} - 3}{h}$$

$$= \lim_{h \to 0} \frac{\frac{3 - 3(1+h)}{1+h}}{h} = \lim_{h \to 0} \frac{-3h}{h(1+h)} = \lim_{h \to 0} \frac{-3}{1+h} = -3$$

So the slope of the tangent line to $y = f(x)$ at $x = 2$

is -3.

b) $f(2) = \frac{3}{2-1} = 3.$

An equation of the tangent line to $y = f(x)$ at $x = 2$ is:

$$y = -3(x-2) + 3 = -3x + 9$$