King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 201

Exam II, 2011 (102)

102 **Duration: 120 minutes**

Key

Name: ______________________

Id: ______________________

Section: ______________________

Answer the questions in the space provided. You must ~~show your~~ work or explain your solution otherwise points may be deducted. If you make an unnecessary approximation in your solution to a problem, your answer will be judged on its accuracy. Points may be deducted for poor or inappropriate approximation.

1. Write clearly.
2. Show all your steps.
3. No credits will be given to wrong steps.
4. Calculators and mobile phones are NOT allowed in this exam.

Q#	Marks	Maximum Marks
1		18
2		15
3		18
4		19
5		20
6		10
Total		100

Problem 1. (18pts)

a) Find an equation in x, y, z for the plane that passes through the point $(1, 2, -3)$ and is perpendicular to the line

$$x = 1 + 2t, \quad y = 2 + t, \quad z = -3 - 5t.$$

A normal vector to the plane is

$\vec{n} = \langle 2, 1, -5 \rangle$. 4 pts

The equation of the plane is

$2(x-1) + (1)(y-2) - 5(z+3) = 0$ 5pts

$\Longleftrightarrow$ $2x + y - 5z = 19$

Give 2pts for the formula and 3pts for the rest

b) Find a set of parametric equations for the line that passes through $P(-1, 2, -3)$ and is perpendicular to the xz-plane.

A direction vector of the line is

$\vec{u} = \langle 0, 1, 0 \rangle$ 3 pts

The parametric equations are

$$\begin{cases} x = -1 & (1+1)\text{ pts} \\ y = 2 + t & (1+1)\text{ pts} \\ z = -3 & (1+1)\text{ pts} \end{cases}$$

Give 1 pt for the general formula (for each)

Problem 2. (15pts)

a) Find the distance from the point $\mathbf{P(2,5,0)}$ to the plane $\mathbf{x+z-7=0}$.

$\vec{n} = \langle 1, 0, 1 \rangle$. $P(x_1, y_1, z_1) = (2, 5, 0)$

$$D = \frac{|x_1 + z_1 - 7|}{|\vec{n}|}$$ 1pt

$$D = \frac{|2 + 0 - 7|}{\sqrt{2}}$$ 2pts

$$= \frac{5}{\sqrt{2}}$$ 2pts

b) For each of the equations below, identify the graph as :

cylinder, hyperboloid of one sheet, cone, sphere, ellipsoid, hyperboloid of two sheets, elliptic paraboloid, hyperbolic paraboloid.

4pts The graph of $\mathbf{2x^2 - 4x + y^2 + 2y - z^2 + 8 = 0}$ is a Hyperboloid of two sheets

2pts The graph of $\mathbf{x = z^2 - y^2}$ is a hyperbolic Paraboloid

2pts The graph of $\mathbf{z = 4 - y^2}$ is a Cylinder

2pts The graph of $\mathbf{y = x^2 + z^2}$ is an Elliptic Paraboloid

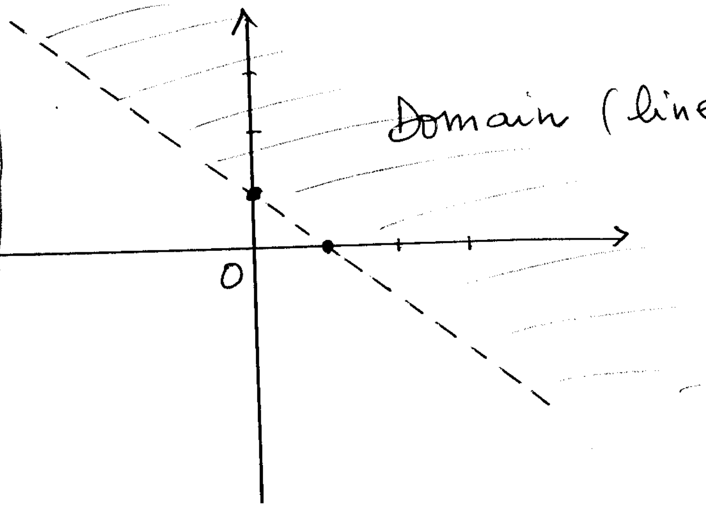
Problem 3. (18pts)

a) Find and sketch the domain of the function

$$\mathbf{f(x,y) = \ln(x + y - 1)}$$

$x + y - 1 > 0$.

$D = \{ (x,y) \mid x + y > 1 \}$ 4pts



Domain (line excluded)

4pts

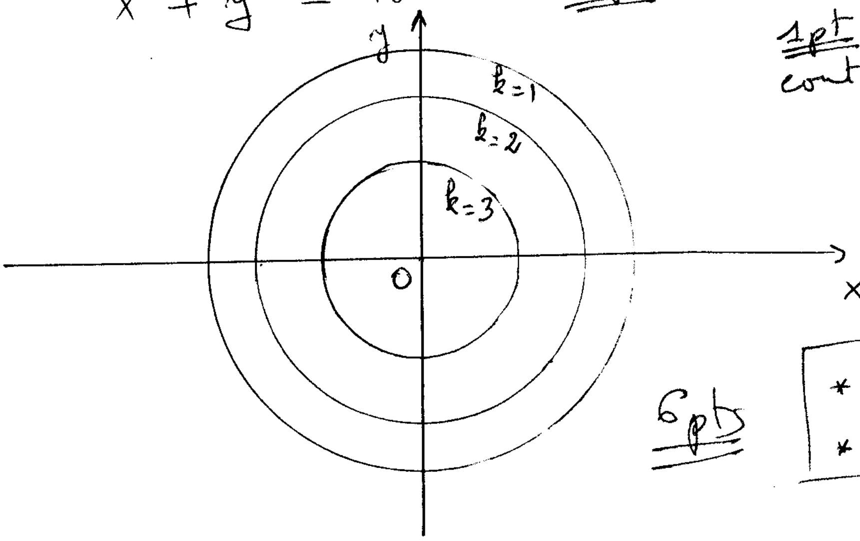
* 1 pt for excluding line
* 1 pt for Correct line
* 2 pts for region

b) Let $\mathbf{f(x,y) = \sqrt{16 - x^2 - y^2}}$. Draw a contour map of level curves $\mathbf{f(x,y) = k}$ with $\mathbf{k = 1; 2; 3}$. Label the level curves by the corresponding values of $\mathbf{k}$.

$f(x,y) = k \iff \sqrt{16 - x^2 - y^2} = k \overset{1pt}{\iff}$

$x^2 + y^2 = 16 - k^2$ 2pts

1pt for the contour map



6pts

* 1pt for each label
* 1pt for each circle

Problem 4. (19pts)

a) Consider the function

$$f(x,y) = \frac{7x^3y}{2x^4+y^4}.$$

Does the limit $\lim_{(x,y)\to(0,0)} f(x,y)$ exist? Why or why not?

3pts * Along the y-axis $(x=0,\ y\neq 0)$, $f(x,y)=0$.

So $\lim_{(x,y)\to(0,0)} f(x,y) = 0$ along the y-axis

3pts * Along the line $y=x$, $f(x,y) = \frac{7x^4}{3x^4} = \frac{7}{3}$.

So $\lim_{(x,y)\to(0,0)} f(x,y) = \frac{7}{3}$ along the line $y=x$

3pts * We conclude that the limit does not exist.

b) Let f be a function of two variables defined by

$$f(x,y) = \begin{cases} \frac{e^{x^2+y^2}-1}{x^2+y^2} & \text{if } (x,y)\neq(0,0) \\ m & \text{if } (x,y)=(0,0). \end{cases}$$

Find the value of m so that f is continuous at $(0,0)$.

Hint: you may use polar coordinates

Using polar Coordinates $x = r\cos\theta$, $y = r\sin\theta$

We have

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{r\to 0} \frac{e^{r^2}-1}{r^2} \quad \text{2pts}$$

$$= \lim_{r\to 0} \frac{2r\,e^{r^2}}{2r} = 1 \quad \text{4pts}$$

f is continuous at $(0,0)$ if and only if

$$\lim_{(x,y)\to(0,0)} f(x,y) = f(0,0) \iff m = 1$$

3pts 1pt

Problem 5. (20pts)

a) If $\mathbf{f(x,y) = xy^2 + \sin(xy)}$, find $\frac{\partial \mathbf{f}}{\partial \mathbf{x}} + \frac{\partial^2 \mathbf{f}}{\partial \mathbf{x} \partial \mathbf{y}}$ at $\mathbf{(x,y) = (0,1)}$.

$$\frac{\partial f}{\partial x} = y^2 + y\cos(xy) \quad , \quad \frac{\partial^2 f}{\partial y \partial x} = 2y - yx\sin(xy) + \cos(xy)$$

4pts   4pts

$$\left. \frac{\partial f}{\partial x} + \frac{\partial^2 f}{\partial y \partial x} \right|_{(x,y)=(0,1)} = 2 + 3$$

$$= 5$$

2pts

b) Find an equation of the tangent plane to the surface

$$\mathbf{z = \sin^2(xy)}$$

at $\mathbf{(x,y) = (\frac{\pi}{4}, 1)}$.

The equation of the tangent line is given by

$$Z = f\left(\tfrac{\pi}{4}, 1\right) + f_x\left(\tfrac{\pi}{4}, 1\right)\left(x - \tfrac{\pi}{4}\right) + f_y\left(\tfrac{\pi}{4}, 1\right)(y-1)$$

$$\Leftrightarrow \quad Z = \underbrace{\frac{1}{2}}_{2\text{pts}} + \underbrace{(1)\left(x - \frac{\pi}{4}\right)}_{3\text{pts}} + \underbrace{\frac{\pi}{4}(y-1)}_{3\text{pts}}$$

* 2pts for formula

$$\Leftrightarrow \quad Z = \frac{1}{2} - \frac{\pi}{2} + x + \frac{\pi}{4} y$$

Problem 6. (10pts)

Let $g(x, y) = ye^x$. Estimate $g(0.1, 1.9)$ using the linear approximation of $g(x, y)$ at $(x, y) = (0, 2)$.

The linearization of g at $(0,2)$ is

$$L(x,y) = g(0,2) + g_x(0,2)(x-0) + g_y(0,2)(y-2)$$

$$= \underbrace{2}_{2pts} + \underbrace{2x}_{3pts} + \underbrace{(1)(y-2)}_{3pts}$$

$$= 2x + y$$

Near $(0, 2)$ $g(x,y) \approx 2x + y$.

Because $(0.1, 1.9)$ is near $(0, 2)$

$$g(0.1, 1.9) \approx 2(0.1) + 1.9$$

$$= 2.1 \quad 2pts$$