MATH 202.10 (sem. 102)

Quiz 1 (Chap. 1.1-2.2) Duration: 20mn

Name: ID number:

1.)(2pts)Give the domain of definition of the function $f(x,y)=\sqrt{9-x^2-y^2}$.

2.)(4pts)Do the following IVP have unique solutions?

a.) $\begin{cases} y'=\sqrt{9-x^2-y^2} \\ y(0)=0, \end{cases}$ b.) $\begin{cases} y'=\sqrt{9-x^2-y^2} \\ y(3)=0. \end{cases}$

3.)(4pts)Solve the separable DE: $(x-3)(y+1)dy=(x+2)(y-1)dx$.

Solution

1) $D_f=\{(x,y)\in\mathbb{R}^2 / \ x^2+y^2\leq 9\}$

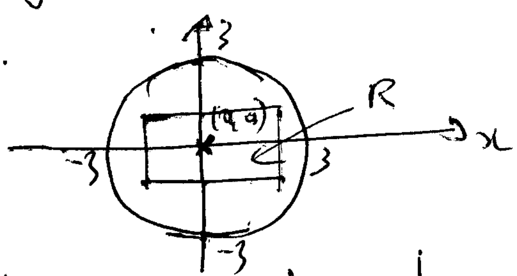
To see this we solve

$9-x^2-y^2\geq 0 \Rightarrow x^2+y^2\leq 9$

2) $f(x,y)=\sqrt{9-x^2-y^2}$, $(x,y)\in D_f$

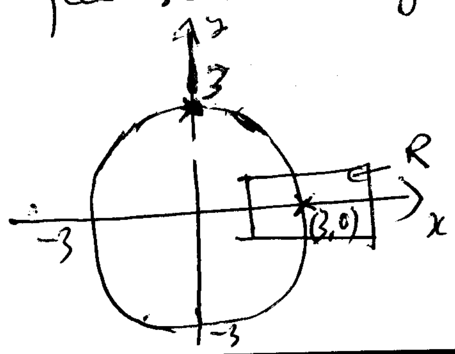
$\frac{\partial f}{\partial y}(x,y)=\frac{-2y}{2\sqrt{9-x^2-y^2}}$, $(x,y)\in D_f$

a)



Let R a rectangular domain that contains $(0,0)$. Both functions f and $\frac{\partial f}{\partial y}$ are continuous on R. Therefore, the IVP $\begin{cases} y'=f(x,y) \\ y(0)=0 \end{cases}$ has a unique solution $y=y(x)$, $x\in]-3,3[$

b)



Let R be a rectangular domain that contains $(3,0)$. $\frac{\partial f}{\partial y}(x,y)$ is not continuous on the arc $x^2+y^2=9$. Therefore, we can't say whether the IVP $\begin{cases} y'=f(x,y) \\ y(3)=0 \end{cases}$ has a unique solution or not.

3) $\int\frac{x+2}{x-3}\,dx=\int\frac{y+1}{y-1}\,dy$

$\int\left(1+\frac{5}{x-3}\right)dx=\int\left(1+\frac{2}{y-1}\right)dy$

$x+5\ln|x-3|=y+2\ln|y-1|+C$

$$\boxed{\frac{(y-1)^2}{(x-3)^5}=Ce^{x-y}, \quad x<3 \text{ or } x>3, \quad C\neq 0}$$