MATH 202.10 (Term 102)

Quiz 3 (Chap. 4.1-4.6) Duration: 20mn

Name: **ID number:**

1.)(3pts)Knowing that $y_1 = x\cos(\sqrt{2}\ln x)$ **is a solution to the DE**

$$-x^2y'' + xy' - 2y = 0,$$

find a second solution y_2 **linearly independent to** y_1 **by using reduction of order.**

2.)(7pts)Solve the nonhomogeneous linear DE

$$2y'' - y' - y = \cos^2\left(\frac{x}{2}\right).$$

Solution

1.) $y_2 = y_1 \int \frac{e^{-\int P(x)}}{y_1^2}\,dx$

$y'' - \frac{1}{x}y' + \frac{2}{x^2}y = 0$

$e^{-\int P(x)} = e^{\int \frac{dx}{x}} = x, \quad x > 0$

$y_2 = y_1 \int \frac{x}{x^2\cos^2(\sqrt{2}\ln x)}\,dx$

$= y_1 \int \frac{1}{x\cos(\sqrt{2}\ln x)}\,dx$

$= y_1 \frac{1}{\sqrt{2}}\left[\tan(\sqrt{2}\ln x)\right]$

$= \frac{\sqrt{2}}{2}\, x\sin(\sqrt{2}\ln x)$

2.) First, we solve $2y'' - y' - y = 0$

The auxiliary equation is

$2m^2 - m - 1 = 0$

$\Delta = 9, \quad m_1 = -\frac{1}{2}, \quad m_2 = 1$

$y_c = c_1 e^{-x/2} + c_2 e^{x}$

To find y_p, we use annihilator approach. $\cos^2(x/2) = \frac{\cos x + 1}{2}$

$D(D^2+1)(2D^2 - D - 1)y = 0$

$\Rightarrow y = \underbrace{c_1 + c_2\cos x + c_3\sin x}_{y_p} + y_c$

$y_p = A + B\cos x + C\sin x$

$y_p' = -B\sin x + C\cos x$

$y_p'' = -B\cos x - C\sin x$

$-2B\cos x - 2C\sin x - (-B\sin x + C\cos x) - (A + B\cos x + C\sin x) = \frac{\cos x + 1}{2}$

$\begin{cases} -3B - C = 1/2 \\ -3C + B = 0 \\ -A = 1/2 \end{cases} \rightarrow B = -3/20, \quad C = -\frac{1}{20}, \quad A = -1/2$

$$\boxed{y = c_1 e^{-x/2} + c_2 e^{x} - \frac{1}{2} - \frac{3}{20}\cos x - \frac{1}{20}\sin x}$$