MATH 202.10 (Term 102)

Quiz 5 (Chap. 4.2-6.2) Duration: 20mn

Name: **ID number:**

Choose and solve 2 questions only.

1.)(5pts)Given that $y_1 = 1$ **is a solution o the DE** $(x^2-1)y'' + 2xy' + y = 0$, $x > 1$, **find a second solution.**

2.)(5pts)Solve the DE $2y''' - y'' - y = 0$.

3.)(5pts)Find the Cauchy-Euler equation whose auxiliary equation has the roots $m = 1+i$, $m = 1-i$ **and** $m = 1$.

4.)(5pts)Find one solution to the DE $xy'' + y = 0$.

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1) $y'' + \frac{2x}{x^2-1}y' + \frac{1}{x^2-1}y = 0$

$$y_2 = y_1 \int \frac{e^{-\int P(x)dx}}{y_1^2}dx$$

$$e^{-\int P(x)dx} = e^{-2\int \frac{x}{x^2-1}dx} = e^{-\ln(x^2-1)} = \frac{1}{x^2-1}$$

$$y_2 = \int \frac{1}{x^2-1}dx = \int \frac{1}{2}\frac{1}{x-1} - \frac{1}{2}\int \frac{1}{x+1} = \frac{1}{2}\ln\left(\frac{x-1}{x+1}\right).$$

2) $2y''' - y'' - y = 0$

The auxiliary equation is

$2m^3 - m^2 - 1 = 0$

$2m^3 - m^2 - 1 \;\big|\; m-1$

$m^2 - 1 \qquad 2m^2 + m + 1$

$\Delta = 1 - 8 = -7$

$m_2 = \frac{-1-i\sqrt{7}}{4}$, $m_3 = \frac{-1+i\sqrt{7}}{4}$.

$$y = c_1e^{x} + c_2e^{-\frac{1}{4}x}\cos\left(\frac{\sqrt{7}}{4}x\right) + c_3e^{-\frac{1}{4}x}\sin\left(\frac{\sqrt{7}}{4}x\right),$$

where c_1, c_2, c_3 are arbitrary constants

3) $(m-1-i)(m-1+i)(m-1) = 0$

$((m-1)^2+1)(m-1) = 0$

$m^3 - 3m^2 + 4m - 2 = 0$

The Cauchy-Euler is

$ax^3y''' + bx^2y'' + cxy' + dy = 0$

Its auxiliary equation is

$am^3 + (b-3a)m^2 + (2a-b+c)m + d = 0$

$a = 1$, $b - 3a = -3 \Rightarrow b = 0$

$2a - b + c = 4 \Rightarrow c = 2$, $d = -2$

$x^3y''' + 2xy' - 2y = 0$

4.) $x = 0$ is a regular singular point

$p(x) = 0 \Rightarrow a_0 = 0$

$q(x) = x^2\left(\frac{1}{x}\right) = x \Rightarrow b_0 = 0$

$r(r-1) = 0 \Rightarrow r = 0$, $r = 1$

$y = \sum_{n=0}^{\infty} c_nx^n$, $|x| < R$.

$$x\sum_{n=2}^{\infty} c_n n(n-1)x^{n-2} + \sum_{n=0}^{\infty} c_nx^n = 0$$

$$\sum_{k=1}^{\infty} c_{k+1}k(k+1)x^k + \sum_{k=0}^{\infty} c_kx^k = 0$$

$$c_0 + \sum_{k=1}^{\infty}\left[c_{k+1}k(k+1) + c_k\right]x^k = 0$$

$c_0 = 0$, $c_{k+1} = -\frac{c_k}{k(k+1)}$, $k = 1, 2, \dots$

$c_2 = -\frac{c_1}{2}$, $c_3 = -\frac{c_2}{6} = \frac{c_1}{12}$, $c_4 = -\frac{c_3}{12} = -\frac{c_1}{144}$

$$y = c_0 + c_1x + c_2x^2 + c_3x^3 + \cdots$$

$$= c_1x - \frac{c_1}{2}x^2 + \frac{c_1}{12}x^3 - \frac{c_1}{144}x^4 + \cdots$$

$$= c_1\underbrace{\left(x - \frac{x^2}{2} + \frac{x^3}{12} - \frac{x^4}{144} + \cdots\right)}_{y_1}$$