MATH 202.18 (sem. 102)

Quiz 1 (Chap. 1.1-2.2) Duration: 20mn

Name: ID number:

1.)(2pts)Give the domain of definition of the function $f(x,y) = \frac{x-1}{1-y^3}$.

2.)(4pts)Do the following IVP have unique solutions?

a.) $\begin{cases} y' = \frac{x-1}{1-y^3} \\ y(0) = 0, \end{cases}$ b.) $\begin{cases} y' = \frac{x-1}{1-y^3} \\ y(0) = 1. \end{cases}$

3.)(4pts)Solve the separable DE: $\cos x (y+1)^2 dx = \sin x dy$.

Solution

1.) $D_f = \{(x,y) \in \mathbb{R}^2 / y \neq 1\}$

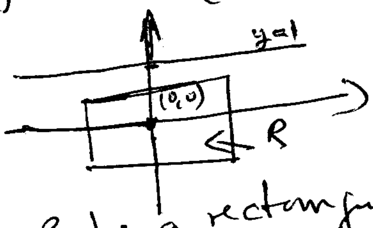
Because $1-y^3 = (1-y)(y^2+y+1)$

and $1-y^3 = 0 \Rightarrow y = 1$, $y^2+y+1 > 0$

2.) $f(x,y) = \frac{x-1}{1-y^3}$, $(x,y) \in D_f$

$\frac{\partial f}{\partial y}(x,y) = \frac{3(x-1)y^2}{(1-y^3)^2}$, $(x,y) \in D_f$

a)

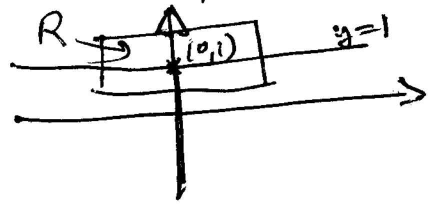


Let R be a rectangular domain that contains (0,0). Both functions f and $\frac{\partial f}{\partial y}$ are continuous on R. Therefore, the IVP

$\begin{cases} y' = f(x,y) \\ y(0) = 0 \end{cases}$

has a unique solution $y = y(x), x \in \mathbb{R}$

b)



Let R be a rectangular domain that contains (0,1). $\frac{\partial f}{\partial y}(x,y)$ is not continuous at $y=1$. Therefore, we can't say whether the IVP

$\begin{cases} y' = f(x,y) \\ y(0) = 1 \end{cases}$

has a unique solution or not.

3.) $\int \frac{dy}{(y+1)^2} = \int \frac{\cos x}{\sin x} dx$

$-\frac{1}{(y+1)} = \ln|\sin x| + C$

or $\boxed{\sin x = c e^{-\frac{1}{y+1}}, y \neq 1}$