MATH 202.18 (Term 102)

Quiz 3 (Chap. 4.1-4.6) Duration: 20mn

Name: **ID number:**

1.)(3pts)Knowing that $y_1 = x^{\frac{1}{4}}\cos(\frac{\sqrt{7}}{4}\ln x)$ **is a solution to the DE**

$$2x^2y'' + xy' + y = 0,$$

find a second solution y_2 **linearly independent to** y_1 **by using reduction of order.**
2.)(7pts)Solve the nonhomogeneous linear DE

$$2y'' + y' - y = \frac{2}{x}.$$

Solution

1.) $y_2 = y_1\int \frac{e^{-\int P(x)}}{y_1^2}\,dx$

$y'' + \frac{1}{2x}y' + \frac{1}{2x^2}y = 0, \quad x > 0$

$\Rightarrow P(x) = \frac{1}{2x} \rightarrow e^{-\int P} = e^{-\frac{1}{2}\int\frac{dx}{x}} = x^{-\frac{1}{2}}$

$y_2 = y_1\int \frac{x^{-\frac{1}{2}}}{x^{\frac{1}{2}}\cos^2(\frac{\sqrt{7}}{4}\ln x)}\,dx$

$= y_1\int \frac{1}{x\cos^2(\frac{\sqrt{7}}{4}\ln x)}\,dx$

$= \frac{4}{\sqrt{7}}y_1\left[\tan(\frac{\sqrt{7}}{4}\ln x)\right]$

$= \frac{4}{\sqrt{7}}x^{\frac{1}{4}}\sin(\frac{\sqrt{7}}{4}\ln x)$

2.) $y = y_c + y_p$

First, we solve $2y'' + y' - y = 0$

The auxiliary equation $2m^2 + m - 1 = 0$

$\Delta = 9, \quad m_1 = -1, \quad m_2 = 1/2$

$y_c = c_1 e^{-x} + c_2 e^{x/2}$

To find y_p we use variation of parameters

$y_p = u_1y_1 + u_2y_2, \quad y_1 = e^{-x}, \quad y_2 = e^{x/2}$

$W = \begin{vmatrix} e^{-x} & e^{x/2} \\ -e^{-x} & \frac{e^{x/2}}{2} \end{vmatrix} = \frac{3}{2}e^{-x/2}$

$u_1' = -\frac{2}{3}\frac{e^{x}}{x} \Rightarrow u_1 = -\int_{x_0}^{x}\frac{2}{3}\frac{e^{t}}{t}\,dt$

$u_2' = \frac{2}{3}\frac{e^{-x/2}}{x} \Rightarrow u_2 = \int_{x_0}^{x}\frac{2}{3}\frac{e^{-t/2}}{t}\,dt$

$y_p = -\frac{2e^{-x}}{3}\int_{x_0}^{x}\frac{e^{t}}{t}\,dt + \frac{2e^{x/2}}{3}\int_{x_0}^{x}\frac{e^{-t/2}}{t}\,dt$

$\Rightarrow y = c_1e^{-x} + c_2e^{x/2} + y_p$