MATH 202.18 (Term 102)

Quiz 4 (Chap. 6.1-6.2) Duration: 20mn

**Name:** **ID number:**

**1.)(3pts)What are the indicial roots at the singular point $x = 0$ of the DE**

$$x^2y'' + 3x\cos xy' + \sin xy = 0,$$

**knowing that** $\cos x = 1 - \frac{1}{2}x^2 + \ldots$ **and** $\sin x = x - \frac{1}{6}x^3 + \ldots$?

**2.)(7pts)Find** $c_1$, $c_2$ **and** $c_3$ **of the solution** $y = \sum_{n=0}^{\infty} c_n x^{n+\frac{3}{2}}$ **to the DE**

$$x^2y'' + xy' - \frac{9}{4}(x+1)y = 0.$$

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1) The normal form of the DE is

$y'' + \frac{3}{x}\cos x\, y' + \frac{\sin x}{x^2} y = 0.$

let $p(x) = x\left(\frac{3}{x}\cos x\right) = 3\cos x$

$p(x) = 3 - \frac{3}{2}x^2 + \ldots \Rightarrow a_0 = 3$

$q(x) = x^2\left(\frac{\sin x}{x^2}\right) = \sin x$

$= x - \frac{x^3}{6} + \ldots \Rightarrow b_0 = 0$

The indicial equation at $x=0$ is $r(r-1) + a_0 r + b_0 = 0$

$\Rightarrow r(r-1) + 3r = 0$

$r(r+2) = 0$

$r = 0$, $r = -2$

2.) $y = \sum_{n=0}^{\infty} c_n x^{n+3/2}$

$y' = \sum_{n=0}^{\infty} c_n (n+\frac{3}{2}) x^{n+\frac{1}{2}}$

$y'' = \sum_{n=0}^{\infty} c_n (n+\frac{3}{2})(n+\frac{1}{2}) x^{n-\frac{1}{2}}$

We substitute into the DE, we find

$\sum_{n=0}^{\infty} c_n (n+\frac{3}{2})(n+\frac{1}{2}) x^{n+\frac{3}{2}} + \sum_{n=0}^{\infty} c_n (n+\frac{3}{2}) x^{n+3/2} - \frac{9}{4}\sum_{n=0}^{\infty} c_n x^{n+1+\frac{3}{2}}$

$- \frac{9}{4}\sum_{n=0}^{\infty} c_n x^{n+3/2} = 0$

$\sum_{n=0}^{\infty} c_n\, n(n+3) x^{n+3/2} - \frac{9}{4}\sum_{n=0}^{\infty} c_n x^{n+1+3/2} = 0$

$\sum_{k=0}^{\infty} c_k\, k(k+3) x^{k+3/2} - \frac{9}{4}\sum_{k=1}^{\infty} c_{k-1} x^{k+3/2} = 0$

$x^{3/2}\left(\sum_{k=1}^{\infty}\left[c_k k(k+3) - \frac{9}{4}c_{k-1}\right]x^k\right) = 0$

$\Rightarrow c_k k(k+3) - \frac{9}{4}c_{k-1} = 0$, $k = 1, 2$

$c_k = \frac{9}{4}\frac{c_{k-1}}{k(k+3)}$, $k = 1, 2, 3 \ldots$

$c_1 = \frac{9}{4}\frac{c_0}{4} = \frac{9}{16}c_0$

$c_2 = \frac{9}{4}\frac{c_1}{10} = \frac{9^2}{4^3 \cdot 10}c_0$

$c_3 = \frac{9}{4}\frac{c_2}{18} = \frac{9^{\cdot}}{4^{4}\cdot 0}c_0$