MATH 202.18 (Term 102)

Quiz 5 (Chap. 4.2-6.2) Duration: 20mn

Name: **ID number:**

Choose and solve 2 questions only.

1.)(5pts)Given that $y_1 = 1$ **is a solution o the DE** $\sin x y'' - 2\cos x y' + y = 0$, $0 < x < \frac{\pi}{2}$, **find a second solution.**

2.)(5pts)Find the linear DE with constant coefficients whose auxiliary equation has the roots $m = 1 + i$, $m = 1 - i$ **and** $m = 1$.

3.)(5pts)Solve the Cauchy-Euler equation $2x^3y''' + 7x^2y'' + 2xy' - 2y = 0$.

4.)(5pts)Solve the DE $(x^2 + 1)y'' + xy' - y = 0$.

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1) $y'' - \frac{2\cos x}{\sin x} y' + \frac{1}{\sin x} y = 0$

$$y_2 = y_1 \int \frac{e^{-\int P(x)dx}}{y_1^2}\,dx$$

$$e^{-\int P(x)dx} = e^{2\int \frac{\cos x}{\sin x}dx} = e^{2\ln(\sin x)} = \sin^2 x$$

$$y_2 = \int \sin^2 x\,dx = \int \frac{1-\cos 2x}{2} = \frac{x}{2} - \frac{\sin 2x}{4}$$

2) $(m-1-i)(m-1+i)(m-1) = 0$

$((m-1)^2+1)(m-1) = 0$

$m^3 - 3m^2 + 4m - 2 = 0$

$$y''' - 3y'' + 4y' - 2y = 0$$

3) $2x^3y''' + 7x^2y'' + 2xy' - 2y = 0$

$2m(m-1)(m-2) + 7m(m-1) + 2m - 2 = 0$

$2m^3 + m^2 - m - 2 = 0$

$2m^3 + m^2 - m - 2 \;\big|\; m-1$

$3m^2 - m - 2 \qquad \underbrace{2m^2 + 3m + 2}$

$\Delta = 9 - 16 = -7$

$m = \frac{-3-i\sqrt{7}}{4}$, $m = \frac{-3+i\sqrt{7}}{4}$

$$y = c_1 x + c_2 x^{-3/4}\cos\left(\frac{\sqrt{7}}{4}\ln x\right) + c_3 x^{-3/4}\sin\left(\frac{\sqrt{7}}{4}\ln x\right),$$

where c_1, c_2, c_3 are arbitrary constants.

4) $x = 0$ is an ordinary point of the DE

$y = \sum_{n=0}^{\infty} c_n x^n$, $|x| < 1$.

$$(x^2+1)\sum_{n=2}^{\infty} c_n n(n-1)x^{n-2} + x\sum_{n=1}^{\infty} nc_n x^{n-1} - \sum_{n=0}^{\infty} c_n x^n = 0$$

$$\sum_{n=2}^{\infty} c_n n(n-1)x^n + \sum_{n=2}^{\infty} c_n n(n-1)x^{n-2} + \sum_{n=1}^{\infty} c_n n x^n - \sum_{n=0}^{\infty} c_n x^n = 0$$

$$\sum_{k=2}^{\infty} c_k k(k-1)x^k + \sum_{k=0}^{\infty} c_{k+2}(k+1)(k+2)x^k + \sum_{k=1}^{\infty} c_k k x^k - \sum_{k=0}^{\infty} c_k x^k = 0$$

$$2c_2 - c_0 + 6c_3 x + \sum_{k=2}^{\infty}\left[c_{k+2}(k+1)(k+2) + c_k(k^2-1)\right]x^k = 0$$

$2c_2 - c_0 = 0 \Rightarrow c_2 = c_0/2$

$6c_3 = 0 \Rightarrow c_3 = 0$

$c_{k+2} = \frac{-(k-1)}{k+2}c_k$, $k = 2, 3, \ldots$

$c_4 = -\frac{1}{4}c_2 = -\frac{c_0}{8}$, $c_5 = c_7 = \cdots = 0$

$$y = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots$$

$$= c_1 x + c_0 + \frac{c_0}{2}x^2 - \frac{c_0}{8}x^4 + \cdots$$

$$= \underbrace{c_1 x}_{y_1} + c_0\underbrace{\left(1 + \frac{x^2}{2} - \frac{x^4}{8} + \cdots\right)}_{y_2}$$

$y = Ay_1 + By_2$, where A, B are arbitrary constants.