# Math 302 Exam I

**Semester (102)** March 10, 2011  Time: 12:30 - 2:00 pm

Name: .................................................................

I.D: .................................  Section: ......

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<tr>
<th>Problem</th>
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<td><strong>Total</strong></td>
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Problem 1. Let \( S = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 \mid x_1 x_2 = x_3 x_4\} \). Check the three conditions of subspace for \( S \) and conclude whether \( S \) is a subspace or not.
Problem 2. Determine whether the vectors \( V_1 = (1, -1, 2, 0) \), \( V_2 = (2, 1, 0, 3) \), \( V_3 = (1, -1, 2, 1) \) are linearly independent or dependent in \( \mathbb{R}^4 \).
**Problem 3.** Let $a, b \in \mathbb{R}$. Consider the following system of linear equations

\[
\begin{align*}
  (\star) \quad \begin{cases}
    x - 2y + 3z &= 4 \\
    2x - 3y + az &= 5 \\
    3x - 4y + 5z &= b
  \end{cases}
\end{align*}
\]

Let $A$ be the matrix of coefficients of the system $(\star)$ and $B = \begin{pmatrix} 4 \\ 5 \\ b \end{pmatrix}$.

(a) Show that the augmented matrix $[A : B]$ is row-equivalent to

\[
\begin{pmatrix}
  1 & 0 & 2a - 9 & : & -2 \\
  0 & 1 & a - 6 & : & -3 \\
  0 & 0 & -2a + 8 & : & b - 6
\end{pmatrix}
\]

(b) Show that for $a = 3$, the system $(\star)$ is consistent for any value of $b$.

(c) Suppose that $a = 4$. Find all values of $b$ for which the system $(\star)$ is inconsistent (has no solution).
Problem 4. Let

\[ A = \begin{pmatrix} 1 & 0 & \alpha \\ 3 & -1 & 0 \\ -2 & 0 & 1 \end{pmatrix}, \]

where \( \alpha \) is a real number.

(a) Find the eigenvalues of \( A \).
(b) Find all values of \( \alpha \) for which \( A \) has repeated eigenvalues.
(c) For the case of positive repeated eigenvalues, find the corresponding eigenvectors.