

# King Fahd University of Petroleum and Minerals

Final Exam for Math 513

Semester 2, Academic year 2010-2011

**Time allowed: 2 hours and 30 minutes**

Full Name: .....

ID Number: .....

**Note:** Show all your work and write clear steps

**Question 1** Use Cramer's rule to solve

$$\begin{cases} 2x + z = 1 \\ y - z = 0 \\ 3x + y = 2 \end{cases}$$

**Question 2** Use the method of separation of variables to solve:

$$\begin{cases} u_t = 4u_{xx}, & 0 < x < 1, & t > 0 \\ u(0, t) = u(1, t) = 0, & t > 0 \\ u(x, 0) = \sin(\pi x) - 3\sin(5\pi x) + 2\sin(7\pi x), & 0 < x < 1 \end{cases}$$

**Question 3** Solve the following system of differential equations:

$$\begin{cases} x' = 3x + z \\ y' = x - y \\ z' = -x + 4y - z \end{cases}$$

**Question 4** Let  $A$  be a square invertible matrix and  $A^T$  be its transpose

a) Show that  $(A^{-1})^T = (A^T)^{-1}$

b) If  $A$  is a  $3 \times 3$  matrix with  $AA^T = 2A$ , then find  $\det(A)$

**Question 5** Use Laplace transform to solve:

$$\begin{cases} u_{tt} = 9u_{xx}, & 0 < x < L, & t > 0 \\ u(0, t) = u(L, t) = 0, & t > 0 \\ u(x, 0) = u_t(x, 0) = 1, & 0 < x < L \end{cases}$$

**Question 6** Find the eigenvalues and eigenfunctions of the Sturm-Liouville problem  $(xy')' + \frac{\lambda}{x}y = 0$  for  $1 < x < e$  subject to  $y'(1) = y'(e) = 0$

**Question 7** Solve the Laplace equation:

$$u_{xx} + u_{yy} = 0, \quad 0 < x, y < 1$$

subject to the following boundary conditions:

a)  $u_y(x, 0) = u(x, 1) = 0$  and  $u(0, y) = u(1, y) = 1$

b)  $u_y(x, 1) = u_x(1, y) = 0$  and  $u(0, y) = u(x, 0) = 1$