King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics
MATH 101 – Calculus I
EXAM I
2010-2011 (103)

Tuesday, July 12, 2011 **Allowed Time: 2 Hours**

Name: ____

ID Number: ____________________ **Serial Number:** ______________

Section Number: ____________ **Instructor's Name:** ________________

Instructions:

1. Write neatly and legibly. You may lose points for messy work.

2. **Show all your work.** No points for answers without justification.

3. **Calculators and Mobiles are not allowed.**

4. Make sure that you have 13 different problems (9 pages + cover page).

Problem No.	Points	Maximum Points
1		12
2		16
3		6
4		4
5		12
6		7
7		5
8		6
9		7
10		8
11		7
12		4
13		6
Total:		100

1. (a) (8 points) From the graph of f, state the numbers at which f is discontinuous and explain why.

f is discontinuous at $x = -4, -2, 2$ and 4.

① point

For $x = -4$: f is discontinuous since $f(-4)$ is undeffined. ① point

① point

$x = -2$: f is discontinuous since $\lim_{x \to -2} f(x)$ DNE. ① Point

① point

$x = 2$: f is discontinuous since $\lim_{x \to 2} f(x)$ DNE. ① point

① point

$x = 4$: f is discontinuous, since $\lim_{x \to 4} f(x)$ DNE. ① point

(b) (4 points) For each of the numbers stated in part (a), determine whether f is continuous from the right, or from the left, or neither.

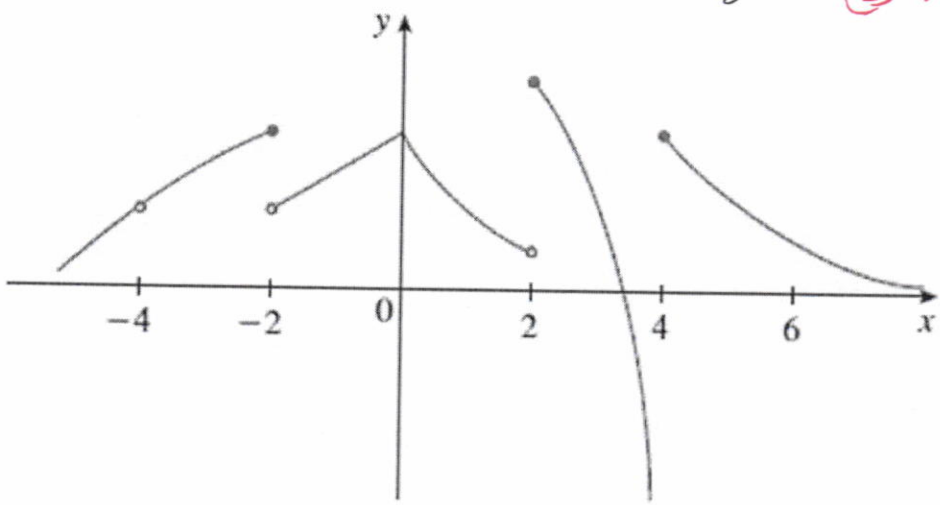
For $x = -4$: f is discontinuous from left and right since

$\lim_{x \to -4^+} f(x) \neq f(4)$ and $\lim_{x \to -4^-} f(x) \neq f(4)$. ① point

$x = -2$: f is continuous from the Left ① Point

$x = 2$: f is continuous from the right. ① point

$x = 4$: f is continuous from the right ① point



2. Evaluate the limit, if it exists:

(a) (3 points) $\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{x^2+x}\right) = \lim_{x\to 0} \frac{x+1-1}{x(x+1)}$ ① point

$= \lim_{x\to 0} \frac{1}{x+1} = 1$ ① point ① point

(b) (3 points) $\lim_{x\to 0^-}\frac{|x|+[[x]]}{x} = \lim_{x\to 0^-} \frac{-x-1}{x} = \infty$ ① point ① point ① point

(c) (4 points) $\lim_{x\to 0}\sin^2 x \cdot \cos\left(\frac{e^x}{x}\right)$.

$-1 \le \cos\left(\frac{e^x}{x}\right) \le 1$ ① point

$-\sin^2 x \le \sin^2 x \cos\left(\frac{e^x}{x}\right) \le \sin^2 x$ ① point

since $\lim_{x\to 0} -\sin^2 x = \lim_{x\to 0} \sin^2 x = 0$, then by using (squeez theorem) ① point

$\lim_{x\to 0} \sin^2 x \cos\left(\frac{e^x}{x}\right) = 0$ ① point

(d) (3 points) $\lim_{x\to 0}\frac{\sin^3 x}{\cos^2 x-1}$. ① point

$= \lim_{x\to 0} \frac{\sin^3 x}{-\sin^2 x} = \lim_{x\to 0} -\sin x = 0$ ① point ① point

(e) (3 points) $\lim_{x\to\frac{\pi}{2}^+} e^{\tan x}+\ln(x-\frac{\pi}{2})$.

$x \to \frac{\pi}{2}^+ \Rightarrow \tan x \to -\infty \Rightarrow e^{\tan x} \to 0$ ① point

$x \to \frac{\pi}{2}^+ \Rightarrow x-\frac{\pi}{2} \to 0^+ \Rightarrow \ln(x-\frac{\pi}{2}) \to -\infty$ ① point

So $\lim_{x\to\frac{\pi}{2}^+} e^{\tan x} + \ln(x-\frac{\pi}{2}) = -\infty$ ① point

3. (6 points) Find numbers a and b such that

$$\lim_{x\to 1} \frac{\sqrt{a\,x+b}-2}{x-1} = 2.$$

(i) $\lim\limits_{x\to 1} \sqrt{ax+b} - 2 = \lim\limits_{x\to 1} \dfrac{\sqrt{ax+b}-2}{x-1} \cdot (x-1) = 2 \cdot 0 = 0$

then $\sqrt{a+b} = 2 \Rightarrow a+b=4 \Rightarrow \boxed{b = 4-a} \longrightarrow (1)$ (1) points

(ii) $\lim\limits_{x\to 1} \dfrac{\sqrt{ax+b}-2}{x-1} \cdot \dfrac{\sqrt{ax+b}+2}{\sqrt{ax+b}+2}$ (1) point

$= \lim\limits_{x\to 1} \dfrac{ax+b-4}{(x-1)(\sqrt{ax+b}+2)}$

$= \lim\limits_{x\to 1} \dfrac{ax+4-a-4}{(x-1)(\sqrt{ax+b}+2)}$ (By using Eq. 1) (1) point

$= \lim\limits_{x\to 1} \dfrac{a(x-1)}{(x-1)(\sqrt{ax+b}+2)} = \lim\limits_{x\to 1} \dfrac{a}{(\sqrt{ax+b}+2)}$ (1) point

$= \dfrac{a}{\sqrt{a+b}+2} = \dfrac{a}{4} = 2 \Rightarrow \boxed{a=8}$ and $\boxed{b=-4}$

(By using Eq. (1)) (1) point (1) point

4. (4 points) Determine whether the function

$$f(x) = \frac{\sqrt{2x+1}-\sqrt{x+1}}{x}$$

has a removable discontinuity, a jump discontinuity, or an infinite discontinuity at $x=0$.

$\lim\limits_{x\to 0} \dfrac{\sqrt{2x+1}-\sqrt{x+1}}{x} \cdot \dfrac{\sqrt{2x+1}+\sqrt{x+1}}{\sqrt{2x+1}+\sqrt{x+1}}$ (1) point

$= \lim\limits_{x\to 0} \dfrac{x}{x(\sqrt{2x+1}+\sqrt{x+1})} = \lim\limits_{x\to 0} \dfrac{1}{\sqrt{2x+1}+\sqrt{x+1}}$ (1) point

$= \dfrac{1}{2}$ (1) point

then $f(x)$ has a removable discontinuity. (1) point

5. (12 points) Use limits to find all vertical and horizontal asymptotes of the graph of $y = \dfrac{x-3}{\sqrt{x^2-4}}$.

(Justify your answer)

(a) $x = 2$ is a vertical asymptote since ① point

$$\lim_{x \to 2^+} \frac{x-3}{\sqrt{x^2-4}} = -\infty \quad \text{① point}$$

① point

(b) $x = -2$ is a vertical asymptote since ① point

$$\lim_{x \to -2^-} \frac{x-3}{\sqrt{x^2-4}} = -\infty \quad \text{① point}$$

① point

(c) $y = 1$ is a horizontal asymptote since ① point

$$\lim_{x \to \infty} \frac{x-3}{\sqrt{x^2-4}} = \lim_{x \to \infty} \frac{x-3}{|x|\sqrt{1-\frac{4}{x^2}}}$$

① point

$$= \lim_{x \to \infty} \frac{x-3}{x\sqrt{1-\frac{4}{x^2}}} \quad \text{① point}$$

$$= \lim_{x \to \infty} \frac{1-\frac{3}{x}}{\sqrt{1-\frac{4}{x^2}}} = 1$$

(d) $y = -1$ is a horizontal asymptote since ① point

$$\lim_{x \to -\infty} \frac{x-3}{\sqrt{x^2-4}} = \lim_{x \to -\infty} \frac{x-3}{|x|\sqrt{1-\frac{4}{x^2}}}$$

① point

$$= \lim_{x \to -\infty} \frac{x-3}{-x\sqrt{1-\frac{4}{x^2}}} \quad \text{① point}$$

$$= \lim_{x \to -\infty} \frac{1-\frac{3}{x}}{-\sqrt{1-\frac{4}{x^2}}} = -1$$