Math 101
Exam 2
103
August 2, 2011
Net Time Allowed: 120 minutes

MASTER VERSION
1. The curve \( y = x^3 + x^2 - x \) has two horizontal tangents at \( x = a \) and \( x = b \). Then \( a + b = \)

   (a) \( \frac{-2}{3} \)

   (b) \( \frac{1}{3} \)

   (c) 0

   (d) \( \frac{5}{3} \)

   (e) \( \frac{1}{2} \)

2. The velocity of a particle in motion along a line is \( v(t) = \ln |2 - t^2| \) for \( t \) in \([0, \sqrt{2}]\). Find the acceleration when the object is at rest.

   (a) 2

   (b) \(-2\)

   (c) \(-1\)

   (d) 1

   (e) None of these
3. The slope of the normal line to the curve $e^y \sin x = e^x \sin y$
at $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ is equal to

(a) $-1$

(b) 2

(c) 0

(d) $e^{\frac{\pi}{4}}$

(e) $-e^{-\frac{\pi}{4}}$

4. If $f(x) = (x - 1)^{\frac{1}{3}}$, then the equation of the vertical tangent to the graph of $f$ is

(a) $x = 1$

(b) $x = -1$

(c) $x = \frac{1}{3}$

(d) $x = -\frac{1}{3}$

(e) None of these
5. If \( y = \sec^2 x \), then \( y'' = \)

(a) \( 6y^2 - 4y \)

(b) \( -2y^2 + 4y \)

(c) \( 6y^2 + 4y \)

(d) \( -2y^2 - 4y \)

(e) None of these

6. If \( f(x) = \frac{h(x) + x}{x + 1} \), \( f'(1) = \frac{1}{2} \), and \( h(1) = 1 \), then \( h'(1) = \)

(a) 1

(b) 0

(c) \( \frac{1}{2} \)

(d) \( \frac{3}{2} \)

(e) \(-1\)
7. If \( y = \frac{(x - 1)^4(2x - 1)^5}{(3x - 1)^3(10x + 1)^7} \), then \( \frac{dy}{dx} \bigg|_{x=0} = \)

(a) \(-75\)
(b) \(75\)
(c) \(0\)
(d) \(47\)
(e) \(-47\)

8. A particle moves according to the law of motion
\( s(t) = \ln(1 + t^2) \) for \( t \) in \([0, \sqrt{2})\), where \( t \) is measured in seconds and \( s \) in meters. The particle is speeding up when

(a) \(0 < t < 1\)
(b) \(1 < t < \sqrt{2}\)
(c) \(0 < t < \sqrt{2}\)
(d) \(0 \leq t < \sqrt{2}\)
(e) None of these
9. If \( f(x) = \ln(1 - x) \), then \( f^{(2011)}(0) \) is equal to

(a) \(-(2010)!\)
(b) \(2011!\)
(c) \(2012!\)
(d) \(-2009!\)
(e) \(\ln(2011)\)

10. If \((0, \beta)\) is a point on the tangent line to the graph of \( y = -\pi + 4 \tan^{-1}\left(\frac{2}{x}\right) \) at \( x = 2 \), then \( \beta = \)

(a) 2
(b) 3
(c) 4
(d) 5
(e) 6
11. When a stone is dropped into a pool, a circular wave moves out from the point of impact at the rate of $3a$ meter per second ($a > 0$ a real constant). How fast is the area enclosed by the wave increasing when the radius of the wave is $a$ meter?

(a) $6\pi a^2 \text{ m}^2/\text{s}$

(b) $3\pi a^2 \text{ m}^2/\text{s}$

(c) $3\pi a \text{ m}^2/\text{s}$

(d) $2\pi a^2 \text{ m}^2/\text{s}$

(e) $6\pi a \text{ m}^2/\text{s}$

12. If $y = e^z$, $z = \ln u^2$ and $u^2+1 = \tan x$ then $\frac{dy}{dx} \bigg|_{x=\pi/4}$ is

(a) 2

(b) $\frac{1}{2}$

(c) $\frac{1}{\sqrt{2}}$

(d) $\sqrt{2}$

(e) None of these
13. \( \lim_{x \to 0} (1 + 2x)^{\frac{3}{2}} = \)

(a) \( e^6 \)
(b) \( e^3 \)
(c) \( e^{\frac{3}{2}} \)
(d) \( e^{\frac{3}{3}} \)
(e) \( 1 \)

14. If \( y = x^x + 2^x + x^2 \), then \( y' = \)

(a) \( x^x (\ln x + 1) + 2^x \ln 2 + 2x \)
(b) \( x^x (\ln x + 1) + x 2^{x-1} + 2x \)
(c) \( xx^{x-1} + x 2^{x-1} + 2x \)
(d) \( xx^{x-1} + 2^x \ln 2 + 2x \)
(e) \( x^x \ln x + 2^x \ln 2 + 2x \)
15. If linear approximation is used to approximate \( \cos(59^\circ) \), we get 
\[
\cos(59^\circ) \approx a + b \left( \frac{\pi}{180} \right),
\]
then \( 2a + \frac{2}{\sqrt{3}} b \) is equal to

(a) 2

(b) 3

(c) 4

(d) 5

(e) 6

16. \[
\lim_{{x \to 0}} \frac{x - x \cos 3x}{\sin^2 2x} =
\]

(a) 0

(b) 3

(c) \( \frac{3}{2} \)

(d) \( \frac{1}{4} \)

(e) \( \frac{3}{4} \)
17. The radius of a circular disk is given as 5 cm with a maximum error in measurement of 0.1 cm. Using differentials, the maximum error in the calculated area of the disk is

(a) \( \pi \text{ cm}^2 \)

(b) \( 10\pi \text{ cm}^2 \)

(c) \( 0.2\pi \text{ cm}^2 \)

(d) \( 0.5\pi \text{ cm}^2 \)

(e) \( 0.1\pi \text{ cm}^2 \)

18. If the tangent line to the parabola \( y = x^2 - 1 \) is perpendicular to the tangent line of the parabola \( y = ax^2 + 1 \) at each intersection point, then \( a = \)

(a) \( \frac{1}{7} \)

(b) \( \frac{1}{9} \)

(c) \( \frac{1}{7} \)

(d) \( -\frac{1}{9} \)

(e) \( -\frac{1}{8} \)
19. Consider the ellipse \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 2 \). The shaded area formed by the two axes and the tangent line to the ellipse at \((a, b)\), has area

(a) \(2ab\)

(b) \(\frac{ab(1 - a)^2}{a - 2}\)

(c) \(\frac{a^2(1 + b^2)}{2b^2(1 + a^2)}\)

(d) \(\frac{a^2(1 + b^2)}{b^2(1 + a^2)}\)

(e) \(ab\)

20. A right circular cone has a base with radius \(r\), and height \(h\). If the radius is expanding at a rate of 2 mm/hr, while the height is contracting/shrinking at the same rate, then the volume will stay constant if

\[ \text{Hint: } V = \frac{1}{3} \pi r^2 h \]

(a) \(r = 2h\)

(b) \(h = 2r\)

(c) \(r = h\)

(d) \(2h + r = 0\)

(e) \(r^2 = h\)