1. Find the value of b for which

$$\sum_{n=0}^{\infty} e^{nb} = 1 + e^b + e^{2b} + e^{2b} + \cdots = 9$$

(a) $\ln\left(\frac{8}{9}\right)$

(b) $\frac{-e}{9}$

(c) $\ln\left(\frac{1}{9}\right)$

(d) $\ln\left(\frac{9}{8}\right)$

(e) $e \ln\left(\frac{1}{9}\right)$

2. Find the Taylor Series about $a = 0$ for $f(x) = \frac{1}{1+2x^2}$.

(a) $\sum_{n=0}^{\infty} (-1)^n 2^n x^{2n}$

(b) $\sum_{n=0}^{\infty} (-1)^n 2^{2n} x^{2n}$

(c) $\sum_{n=0}^{\infty} 2^n x^{2n}$

(d) $\sum_{n=0}^{\infty} 2^{2n} x^{2n}$

(e) $\sum_{n=0}^{\infty} (-1)^n \left(\frac{x}{2}\right)^{2n}$

3. Find the area of the region bounded by $y = \dfrac{2}{x}$ and $y = -x + 3$.

(a) $\dfrac{3}{2} - \ln 4$

(b) $\dfrac{3}{2} - \ln \sqrt{2}$

(c) $\sqrt{3} - \ln 4$

(d) $\sqrt{3} - \ln \sqrt{2}$

(e) $\sqrt{6} - \ln \sqrt{2}$

4. Compute the following improper integral

$$\int_0^\infty \left(\sum_{n=0}^{\infty} (-1)^n \frac{x^n}{n!} \right) dx.$$

(a) 1

(b) -1

(c) 0

(d) π

(e) The integral diverges

5. Let $f(x) = \sum_{n=0}^{\infty} (-1)^n n^2 x^n$. What is the third derivative $f^{(3)}(x)$, evaluated at $x = 0$?

(a) -54

(b) 0

(c) -9

(d) $-3/2$

(e) $3/2$

6. Evaluate $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{3^{2n}(2n+1)!}$.

(a) $\frac{3\sqrt{3}}{2}$

(b) $\pi e^{-\pi/3}$

(c) $\pi \sinh\left(\frac{\pi^2}{9}\right)$

(d) $3 \sinh\left(\frac{\pi^2}{9}\right)$

(e) $\pi e^{-\pi^2/9}$

7. How many of the following series converge?

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}+1}{n+2} \quad \sum_{n=1}^{\infty} \frac{2^n+n^2}{3^n+n^3} \quad \sum_{n=1}^{\infty} \frac{n^2+3}{(n+4)^2} \quad \sum_{n=3}^{\infty} \frac{\ln(n)}{n^2} \quad \sum_{n=2}^{\infty} \frac{n^2}{\ln(n)}.$$

(a) Two

(b) One

(c) Three

(d) Four

(e) None

8. Find the interval of convergence of $\sum_{n=0}^{\infty} \frac{(-3)^n(x-1)^n}{\sqrt{n+1}}$.

(a) $\left(\frac{2}{3}, \frac{4}{3}\right]$

(b) $\left(\frac{2}{3}, \frac{4}{3}\right)$

(c) $\left[\frac{2}{3}, \frac{4}{3}\right)$

(d) $\left(\frac{1}{3}, \frac{5}{3}\right]$

(e) $\left(\frac{1}{3}, \frac{5}{3}\right)$

9. What is the radius of convergence of the series

$$\sum_{n=1}^{\infty} \frac{2 \cdot 5 \cdot 8 \cdots (3n-1)}{3 \cdot 7 \cdot 11 \cdots (4n-1)} (x+1)^n \ ?$$

(a) $\frac{4}{3}$

(b) $\frac{3}{4}$

(c) $\frac{2}{3}$

(d) $\frac{3}{2}$

(e) ∞

10. Compute $\sum_{n=1}^{\infty} \frac{n}{3^{n-1}}$.

(a) 9/4

(b) 3/2

(c) 2/3

(d) 4/3

(e) 2

11. The series $\sum_{n=1}^{\infty} \frac{(-1)^n(n+3)2^{2n}}{3^{n+100}}$

(a) diverges by the ratio test

(b) converges absolutely by the ratio test

(c) converges conditionally (but not absolutely) by the ratio test

(d) converges absolutely by comparison with $\sum_{n=1}^{\infty} \frac{1}{3^n}$

(e) converges conditionally (not absolutely) by comparison with $\sum_{n=1}^{\infty}(-1)^n \frac{n+3}{3^n}$

12. Compute the Maclaurin series (i.e., the Taylor series about 0) of

$$f(x) = x^2 + \arcsin(x) = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \cdots$$

up to and including terms of order two. Then calculate

$a_0^2 + a_1^2 + a_2^2$.

(a) 2

(b) 3

(c) 5

(d) 1

(e) 4

13. $\int_2^3 \dfrac{x^3 - x^2 + 1}{x^2 - x}\, dx =$

(a) $\dfrac{5}{2} - \ln 3 + 2\ln 2$

(b) $\dfrac{5}{2} + \ln 3 - 2\ln 2$

(c) $\dfrac{1}{2} + \ln 3$

(d) $\dfrac{5}{2} + \ln 3$

(e) $\dfrac{5}{2} - \ln 3$

14. $\int_0^{\pi/2} \cos^3(x)\, dx =$

(a) $2/3$

(b) $1/3$

(c) $1/2$

(d) 0

(e) 1

15. Evaluate the integral $\int_2^5 \frac{1}{4-x}\,dx$.

(a) the integral diverges

(b) $\ln 2$

(c) $-\ln 2$

(d) 0

(e) 1

16. In the Maclaurin series of $\int \frac{e^{x^2}-1}{x}\,dx$, the coefficient of x^6 is

(a) $\frac{1}{36}$

(b) $\frac{1}{48}$

(c) $\frac{1}{18}$

(d) $\frac{1}{8}$

(e) $\frac{1}{6}$