1) If \( \int_{-1}^{0} f(x)dx = \int_{0}^{\pi/4} [2x - \sec^2(x)f(\tan x)]dx \), then find \( \int_{-2}^{2} f \left( \frac{x}{2} \right) dx \).

2) Find the volume of the solid obtained by rotating about the y-axis the region bounded by \( x = 2y^2 - y^3 \) and \( x = 0 \).

Solution:

1) Let \( u = \tan x \), then \( du = \sec^2 x \, dx \) and

\[
\int_{-1}^{0} f(x)dx = \int_{0}^{\pi/4} 2xdx - \int_{0}^{1} f(u)du \Rightarrow \int_{-1}^{0} f(x)dx + \int_{0}^{1} f(u)du = \frac{\pi^2}{16}
\]

Now let

\[
y = \frac{x}{2} \Rightarrow dy = \frac{1}{2} \, dx \text{ and } y = -1, 1
\]

Hence

\[
\int_{-2}^{2} f \left( \frac{x}{2} \right) dx = 2 \int_{-1}^{1} f \, y \, dy = 2 \frac{\pi^2}{16} = \frac{\pi^2}{8}
\]