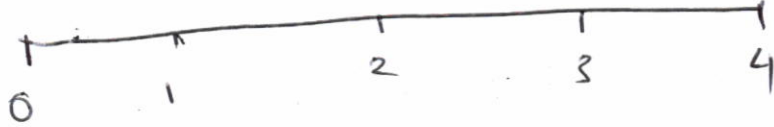


Name: _____

1. For $f(x) = \frac{x}{1+x}$, $0 \leq x \leq 4$. Find L_4 and R_4 to estimate the area under the curve, above the x -axis and between the lines $x = 0$, $x = 4$.

Soln.

$$\Delta x = \frac{b-a}{4} = \frac{4-0}{4} = 1$$

$$\begin{aligned} \Rightarrow \textcircled{1} L_4 &= \Delta x [f(x_0) + f(x_1) + f(x_2) + f(x_3)] \\ &= (1) [f(0) + f(1) + f(2) + f(3)] \\ &= \frac{0}{1+0} + \frac{1}{1+1} + \frac{2}{1+2} + \frac{3}{1+3} \\ &= 0 + \frac{1}{2} + \frac{2}{3} + \frac{3}{4} = \frac{6+8+9}{12} \\ &= \boxed{\frac{23}{12}} \end{aligned}$$

$$\begin{aligned} \textcircled{2} R_4 &= (1) [f(1) + f(2) + f(3) + f(4)] \\ &= \frac{1}{1+1} + \frac{2}{1+2} + \frac{3}{1+3} + \frac{4}{1+4} \\ &= \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} \\ &= \frac{30 + 40 + 45 + 48}{60} = \boxed{\frac{163}{60}} \end{aligned}$$