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1. Write clearly.
2. Show all your steps.
3. No credit will be given to wrong steps.
4. Do not do messy work.
5. Calculators and mobile phones are NOT allowed in this exam.
6. Turn off your mobile.
1a. If \( A^2 = B \) and \( B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \). Find \( A \). \\

1b. If \( A \) is an \( n \times n \) matrix. Show that \( |AA^T| \geq 0 \).
2. Find the value of $a$ such that

$$
\begin{vmatrix}
2 & 1 & 0 \\
0 & -1 & 3 \\
0 & 0 & a
\end{vmatrix} + 
\begin{vmatrix}
0 & a & 1 \\
1 & 3a & 0 \\
-2 & a & 2
\end{vmatrix} = 14
$$

3. Express the matrix $A = \begin{bmatrix} 3 & 6 \\ 1 & 3 \end{bmatrix}$ as a product of elementary matrices.
4. Using Gauss-Jordan method to find a basis and the dimension of the solution space of the homogeneous linear system

\[
\begin{align*}
7x_1 + 2x_2 - 2x_3 - 4x_4 + 3x_5 + 8x_6 &= 0 \\
-3x_1 - 3x_2 + 2x_4 + x_5 - 2x_6 &= 0 \\
4x_1 - x_2 - 8x_3 + 20x_5 - 14x_6 &= 0
\end{align*}
\]
5. Find the matrix \( \text{adj} \ A \) where 
\[
A = \begin{bmatrix}
1 & a & a^2 \\
1 & b & b^2 \\
1 & c & c^2
\end{bmatrix}
\]
6. Consider the following linear system

\[-2x_1 + 3x_2 - x_3 = 1\]
\[x_1 + 2x_2 - x_3 = 4\]
\[-2x_1 - x_2 + x_3 = -3\]

Use Cramer’s Rule to find \(x_3\)?
7. Is $W = \{(a, b, c, d) | c = a + 2b \text{ and } d = a - 3b\}$ subspace of $\mathbb{R}^4$? verify?
8. Let $X_1 = (1, 2, 1, -1)$, $X_2 = (1, 0, 2, -3)$ and $X_3 = (1, 1, 0, -2)$. Write $X = (2, 1, 5, -5)$ as a linear combination of $X_1$, $X_2$ and $X_3$.

9. Let $S = \{X_1, X_2, X_3\}$ where $X_1 = (1, 0, 1, 2)$, $X_2 = (1, 0, 2, -3)$ and $X_3 = (1, 1, 1, 3)$. Is $S$ linearly independent? Do $S$ form a basis for $\mathbb{R}^4$? verify?
10a. Let $y_1 = \cos(\sqrt{2}x)$ and $y_2 = \sin(\sqrt{2}x)$ be two solutions of the DE $y'' + 2y = 0$. Show that $y_1$ and $y_2$ are linearly independent and find the general solution of the given DE.

10b. Find the particular solution to the DE

$$16y'' - 40y' + 25y = 0, \quad y(0) = 3, \quad y'(0) = -\frac{9}{4}$$