

**Practice Exam II: MATH 101-T111**

Q1. Find  $dy/dx, d^2y/dx^2$  when (i)  $y^2 = (\ln 2)^x + x^{\ln 2}$  (ii)  $\tan(x - y) = 1 + xy / (x - y)$   
 (iii)  $\ln|x - y^2| = 2xy$  (Note:  $d \ln|x|/dx = 1/x$ ; use chain rule)

Q2. (a) If  $g(x) = \frac{xk(x) + e^x}{x^2 - 1}$  and  $g'(0) = 5$ , find  $k'(0)$ . (b) If  $y = \frac{3^{\cos^2 s}}{\sin^{-1} x}$ , find  $y'$ .

Q3. Find the sum of the slope of the normal and tangent lines to the curve  $(\cos y) \ln x - xe^y = x$  at  $(\pi, 0)$ .

Q4. Find the an equation of verical tantent line(s) (if exists) for the following functions:

(i)  $y = (3x - 1)^{1/5}$ , (ii)  $y = |x^2 - 1|^{1/2}$ ,  $y = |\ln x|$

(Hint: Check the points where the function takes 0 value(s).)

Q5. (a) Find the values of  $y, w, x$  at  $u = \pi/4$  when  $y = 2w^3 + 1$ ,  $w = \sin x^2$ ,  $x = \sqrt{\pi/4} \tan u$ .



(b) Find  $dy/du|_{u=\pi/4}$  [Ans:  $\pi/4$ ] (Chain Rule).

Q6. The curve  $y = \sin^2(x + (\pi/3))$  has 2 horizontal tangent lines in the interval  $[0, 3\pi/2]$  at  $x = a$  and  $x = b$ . Find the values of  $a$  and  $b$ .

Q7. Find the limits: (i)  $\lim_{x \rightarrow 0} (5 \tan 8x) \sin 3x / (3x^3 \csc 3x)$  (ii)  $\lim_{x \rightarrow \pi/7} \frac{7 \tan 4(x - \pi/7)}{3(x - \pi/7)^2 \csc 4(x - \pi/7)}$

(iii)  $\lim_{x \rightarrow 0} (1 + 7x)^{-6/x}$  (iv)  $\lim_{x \rightarrow \infty} (1 + (2/3x))^9$

Q8. Find  $dy/dx|_{x=1}$  when  $y = \frac{(x+1)(2x-3)\tan^{-1}x}{e^{x-1}(2x-1)^2(x+4)^3}$ .

Q9. The position of a particle is given by (i)  $s(t) = 1 + 2 \cos t$ ,  $[0, 2\pi]$  (ii)  $s(t) = t^3 - 4.5t^2 - 7t$ ,  $[0, 2\pi]$  Find in both cases

- (i) the points when the particle is at rest
- (ii) the displacemen of the particle in the time interval.
- (iii) the distance travelled by thae particle in the time interval
- (iv) the intervals in which the particle is moving in the +ive direction.
- (iv) the time when the particle reach the velocity of 1 m/sec<sup>2</sup> (iv) Draw the diagram to show the motion of particle.

Q10. The velocity of a particle is given by  $v(t) = x^4 - 2x^3/3 - 8x^2 + 16$ . Find the interval in which the particle is speeding up.

Q11. Find  $d^{1103}y/dx^{1103}|_{x=1}$  when (i)  $y = 5/(2x+1)$  (ii)  $y = 5 \sin(\pi x/2)$  (iii)  $y = e^{-3x+2}$  (vi)  $y = 3 \ln(3x+4)$ .

Q12. Using the definiion of the derivative, find  $\lim_{h \rightarrow 0} \frac{\sqrt{\log_2(e+h)} - (1/\sqrt{\ln 2})}{h}$ . [Note: Here,  $f(x) = \sqrt{\log_2 x} = \sqrt{\ln x / \ln 2}$ ;  $x_0 = e$ ]

Q13. Let  $f$  be a differentiable function such that  $f(2) = 2, f(4) = 1, f'(2) = 3, f'(4) = -1$ , If  $G'(x) = f(2x).f(x)$ , then find  $G'(2)$ .

Q14. Define the variables, draw the diagram, find the equation(s) showing their relationship and solve the problems from Exercise 3.9:

Prob: 2, 3, 12, 17, 23, 35

Also, read Examples of sec 3.9

Important formulas to remember: Area and Perimeter (or circumfrand) of Rectangle, Triangle, Circle.

Volume and surface area of cube, box, cylinder, sphere, cone.

Discussing your solution & Office Hrs:

Work out the solutions. You must bring your solution to discuss the difficulty of the problem during my office hours.

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