1. The function \( f(x) = \begin{cases} \frac{ax^2 + bx}{x + a^2} & \text{if } x \leq 1 \\ x + a^2 & \text{if } x > 1 \end{cases} \) is twice differentiable everywhere. Then \( a^2 + b^2 = \)

   a) 1  
   b) 0  
   c) \( \frac{5}{4} \)  
   d) 2  
   e) 5

2. If \( f(x) = (2x - 1)^3 \), then the equation of the vertical tangent to the graph of \( f \) is

   a) \( x = \frac{1}{2} \)  
   b) \( x = -\frac{1}{2} \)  
   c) \( x = \frac{2}{3} \)  
   d) \( x = -\frac{2}{3} \)  
   e) \( x = \frac{4}{3} \)
3. The equations of the horizontal tangents to the curve \( y = x^3 - 3x - 2 \) are

a) \( y = 0 \) and \( y = -4 \)
b) \( y = 1 \) and \( y = -1 \)
c) \( x = 1 \) and \( x = -1 \)
d) \( y = -4 \) and \( y = 1 \)
e) \( y = 0 \) and \( y = -1 \)

4. At how many real values of \( x \) does the curve \( y = x^6 - 3 \, x^2 + x + 5 \) have a tangent line parallel to the line \( y = x \)?

a) 3
b) 1
c) 2
d) 4
e) 5
5. If the tangent line to the graph of \( f(x) = \frac{2x}{2x + 1} \) at the point \((\alpha, \beta)\) is \( y = 2x + 1 \), then \( \beta^2 = \)

   a) 1  
   b) 2  
   c) 3  
   d) 4  
   e) 5

6. If \( f(x) = xe^x \) and \( n \) is a positive integer, then \( f^{(n)}(1) = \)

   a) \((n + 1) e\)  
   b) \(n e\)  
   c) \((n - 1) e\)  
   d) \((n + 2) e\)  
   e) \(n e + 1\)
7. If \( y = \frac{1 + \sin x}{1 + \cos x} \), then \( \frac{dy}{dx} = \)

a) \( \frac{1 + \sin x + \cos x}{(1 + \cos x)^2} \)

b) \( \frac{\sin x + \cos x}{1 + \cos x} \)

c) \( \frac{\sin x + \cos x}{(1 + \cos x)^2} \)

d) \( \frac{1 + \sin x}{(1 + \cos x)^2} \)

e) \( \frac{2}{1 + \cos x} \)

8. \( \lim_{\theta \to 1} \frac{\sin(\theta - 1)}{\theta^2 + \theta - 2} = \)

a) \( \frac{1}{3} \)

b) 0

c) \( \frac{1}{2} \)

d) 2

e) 1
9. If \( y = \sin(x^2) \) and \( x = \cos t \), then \( \frac{dy}{dt} =

\begin{align*}
\text{a)} & \quad - \sin 2t \cos(\cos^2 t) \\
\text{b)} & \quad \sin 2t \cos(\cos^2 t) \\
\text{c)} & \quad - \sin t \cos(\cos^2 t) \\
\text{d)} & \quad - \sin 2t \cos^3 t \\
\text{e)} & \quad \sin 2t \cos^3 t 
\end{align*}

10. Let \( f \) and \( g \) be differentiable functions and \( h(x) = f(x^2 g(x)) \). If \( g(2) = -2 \) and \( g'(2) = 2 \), then \( h'(2) =

\begin{align*}
\text{a)} & \quad 0 \\
\text{b)} & \quad -2 \\
\text{c)} & \quad 2 \\
\text{d)} & \quad 3 \\
\text{e)} & \quad -3 
\end{align*}
11. The equation of the tangent line to the curve given implicitly by
\[ \sqrt{x + y} = y^2 \]

at the point \((0, 1)\) is

a) \(3y - x = 3\)
b) \(2y + x = 1\)
c) \(3y + x = 3\)
d) \(2y - x = 2\)
e) \(2y + x = 3\)

12. The equation of the normal line to the curve \(y = \tan^{-1}(\sqrt{x - 1})\) at \(x = 2\) is

a) \(y = -4x + 8 + \frac{\pi}{4}\)
b) \(y = \frac{1}{4}x - \frac{1}{2} + \frac{\pi}{4}\)
c) \(y = 4x - 8 + \frac{\pi}{4}\)
d) \(y = -\frac{1}{4}x + \frac{1}{2} + \frac{\pi}{4}\)
e) \(y = -4x + 8 - \frac{\pi}{4}\)
13. If \( f(x) = (x^2 + 2x)^{50} \), then \( f^{(100)}(1) = \)

a) 100 !
b) 100
c) 0
d) 3(99 !)
e) 2(50 !)

14. The slope of the tangent line to the graph of \( y = (2x + 1)^{\sin 3x} \) at \( x = \frac{\pi}{6} \) is

a) 2
b) \( 4 \left( \frac{\pi}{3} + 1 \right) \)
c) 6
d) \( 2 \left( \frac{\pi}{3} + 1 \right) \)
e) \( \frac{4}{\frac{\pi}{3} + 1} \)
15. If \( y = \frac{(x + 2)^2(2x - 1)^3}{\sqrt{x + 1}} \), then \( y'(0) = \)

a) 22
b) \(-\frac{11}{2}\)
c) 44
d) 24
e) \(-11\)

16. The position function of a particle moving along a line is

\[ s(t) = \sin t + \cos t \]

where \( t \) is measured in seconds and \( s \) in meters. The total distance traveled by the particle in the interval \([0, \pi]\) is

a) \(2\sqrt{2}\) meters
b) 2 meters
c) 4 meters
d) \(2\sqrt{2} + 2\) meters
e) \(2\sqrt{2} - 2\) meters
17. The position function of a particle moving along a line is

\[ s(t) = t^3 - 6t^2 + 9t \quad (0 \leq t \leq 5). \]

The time interval(s) where the particle is moving forward is (are)

a) (0, 1) and (3, 5)
b) (0, 3)
c) (0, 3) and (4, 5)
d) (1, 3)
e) (0, 2) and (3, 5)

18. The two equal sides of an isosceles triangle have length 4m. If the angle between them is increasing at a rate of 0.06 rad/s, then the rate at which the area of the triangle is changing when the angle between the sides of the triangle is \( \frac{\pi}{3} \) equals

a) 0.24 m²/s
b) −0.24 m²/s
c) 2.4 m²/s
d) −2.4 m²/s
e) 0.024 m²/s
19. If a snow ball melts so that its surface area decreases at a rate of $1 \text{ cm}^2/\text{min}$, then the rate at which the diameter changes, when the diameter is $10 \text{ cm}$ equals

Hint: Surface area of a sphere = $4\pi r^2$

a) $-\frac{1}{20\pi} \text{ cm/min}$

b) $\frac{1}{20\pi} \text{ cm/min}$

c) $-\frac{1}{40\pi} \text{ cm/min}$

d) $\frac{1}{40\pi} \text{ cm/min}$

e) $-\frac{1}{10\pi} \text{ cm/min}$

20. The equation of the tangent line to the graph of $y = \ln x$ and passes through the origin is

a) $e \ y = x$

b) $y = e \ x$

c) $y = \frac{1}{e}(x - 1)$

d) $y = \frac{1}{e}(x + 1)$

e) $y = 2 \ e \ x$