1. If \( \lim_{x \to 0} \frac{3^{ax} - 1}{x} = 1 \), then the value of \( a \) equals

a) \( \frac{1}{\ln 3} \)
b) \( \ln 3 \)
c) 3
d) \(-\ln 3\)
e) \( \frac{1}{3} \)

2. If

\[
f(x) = \begin{cases} 
\frac{\sin(\cos x - 1)}{x} & \text{if } x \neq 0 \\
a & \text{if } x = 0
\end{cases}
\]

The value of \( a \) which makes the function \( f(x) \) continuous everywhere is

a) 0
b) \( \pi \)
c) \(-\pi\)
d) \(-1\)
e) \( \frac{\pi}{2} \)
3. If \( \cosh x = \frac{5}{3} \) and \( x < 0 \), then \( 3 \sinh x - 5 \tanh x \) is

a) 0  
b) \(-8\)  
c) 8  
d) \(-6\)  
e) 6

4. If there are two tangent lines from the point \( P(0, -1) \) that touch the graph of \( f(x) = x^2 \) at \( x = a \) and \( x = b \), then \( f'(a) + f'(b) = \)

a) 0  
b) 2  
c) 4  
d) 8  
e) 16
5. Using the graph of \( f(x) = \tan x \), the maximum value of \( \delta \) such that \( |f(x)| < 0.1 \) whenever \( |x| < \delta \) is equal to

a) \( \tan^{-1}0.1 \)

b) \( \tan^{-1}(-0.1) \)

c) \( \tan^{-1}0.2 \)

d) \( 2\tan^{-1}0.1 \)

e) 0

6. If \( G(x) = 60\sqrt{x} - 162\sqrt{x} \), then the slope of the tangent line to the graph of \( y = G'(x) \) at \( x = 1 \) is

a) 21

b) -18

c) 61

d) -13

e) 28
7. If \( h(x) = \frac{\sec x}{g(x)} \) with \( h'(\pi) = 2 \) and \( g(\pi) = \sqrt{2} \), then \( g'(\pi) \) is

a) 4
b) \( 4 + \sqrt{2} \)
c) \( -2\sqrt{2} \)
d) 5
e) \( 2\sqrt{2} \)

8. If \( f(x) = \sec x \), then \( f''(\frac{\pi}{4}) = \)

a) \( 3\sqrt{2} \)
b) 1
c) \( \frac{12}{13} \)
d) \( \frac{5\sqrt{3}}{4} \)
e) 9
9. The slope of tangent line to the graph of the equation $3x^4 + xy + 3y^4 = 59$ at the point $(1, 8)$ is

a) $-\frac{4}{3}$

b) $\frac{19}{8}$

c) $-\frac{20}{9}$

d) $\frac{16}{7}$

e) $\frac{20}{9}$

10. The slope of the tangent line to the graph of $y = (\ln x)^x$ at the point $(e, 1)$ is

a) 1

b) 2

c) $e$

d) 0

e) $-1$
11. If \( f(x) = \ln \left| \frac{(x^2 + 4)^{5/2}}{(x + \sqrt{x})^{3/2}} \right| \), then \( f'(1) = \)

a) \(-\frac{1}{8}\)  
b) \(\frac{5}{9}\)  
c) \(\frac{7}{8}\)  
d) \(-\frac{4}{9}\)  
e) \(-\frac{13}{8}\)

12. A particle is moving along the curve \( y = \sqrt{x} \). As the particle passes through the point \((4, 2)\), its \(x\)-coordinate increases at a rate of 3 cm/s. The rate of change of the distance from the particle to the origin at that instant is

a) \(\frac{27}{4\sqrt{5}}\)  
b) 9  
c) \(\frac{2}{\sqrt{5}}\)  
d) \(\frac{2}{3\sqrt{7}}\)  
e) \(\frac{4}{\sqrt{11}}\)
13. Using differentials or linear approximation the number $\sqrt{26}$ is estimated as

a) $\frac{80}{27}$

b) $\frac{82}{27}$

c) 2.99

d) 3.1

e) $\frac{85}{27}$

14. The value of $\frac{d}{dx}(\sinh^{-1}(\text{csch}x))$ at $x = \ln 3$ is

a) $-\frac{3}{4}$

b) $-\frac{5}{4}$

c) $\frac{7}{6}$

d) $\frac{11}{6}$

e) $\frac{7}{4}$
15. The position function of a particle moving along a straight line is 
\[ s(t) = 8t - 3t^2 \] for \( t \) in \([1, 2]\), where \( t \) is measured in seconds and \( s \) in meters. The particle is speeding up when

a) \( \frac{4}{3} < t < 2 \)
b) \( 1 < t < \frac{4}{3} \)
c) \( 1 < t < 2 \)
d) \( 1 < t < \frac{3}{2} \)
e) \( \frac{3}{2} < t < 2 \)

16. The radius of a circular disk is measured to be 5 cm with a maximum error in measurement of 0.1 cm. Using differentials, the maximum error in calculating circumference of the circular disk is

a) \( \pi \frac{5}{5} \) cm
b) \( \pi \) cm
c) \( \pi \frac{10}{10} \) cm
d) \( \pi \frac{2}{2} \) cm
e) \( \pi \frac{50}{50} \) cm
17. The sum of the absolute maximum value and the absolute minimum value of the function \( f(x) = 2 \sin x + \cos 2x \) on the internal \([0, \frac{\pi}{2}]\) is

a) \( \frac{5}{2} \)
b) 2
c) \( \frac{3}{2} \)
d) 3
e) \( \frac{7}{2} \)

18. The sum of all critical numbers of the function \( f(x) = \frac{(x - 4)^2}{\sqrt{x + 1}} \) is

a) 2
b) 1
c) 4
d) \(-2\)
e) \(-1\)
19. If \( f(5) = -\frac{5}{2} \) and \( f'(x) \geq -\frac{1}{2} \) for \( 3 \leq x \leq 5 \), then the largest possible value of \( f(3) \) is

a) \( -\frac{3}{2} \)

b) \( \frac{1}{2} \)

c) \( -\frac{1}{4} \)

d) \( -\frac{2}{5} \)

e) 0

20. If \( c \) is a number satisfying the conclusion of the Mean Value Theorem when applied to \( f(x) = \tan^{-1} x \) on \([0, 1]\), then \( \pi c^2 = \)

a) \( 4 - \pi \)

b) 4

c) \( \pi + 1 \)

d) \( 2\pi \)

e) \( \pi - 2 \)
21. The function \( f(x) = \frac{x}{x^2 + 1} \) is increasing on

a) \((-1, 1)\)
b) \((-\infty, -1) \cup (1, \infty)\)
c) \((-\infty, 0)\)
d) \((0, \infty)\)
e) \((-\infty, \infty)\)

22. If the function \( f(x) = x^3 + 2ax^2 - 3bx + 1 \) has an inflection point at \((1, 2)\), then \(2a + b^3\) equals

a) \(-4\)
b) \(-2\)
c) 2
d) 3
e) \(-1\)
23. The value of the limit \( \lim_{x \to 0} \frac{1}{x(1 - \sin x)} \) equals

a) \( \frac{1}{e} \)

b) \( e \)

c) 1

d) \( \frac{1}{\sqrt{e}} \)

e) 0

24. The slant asymptote of \( y = \frac{2x^3 + 3x^2 + 20}{x^2 + 1} \) is

a) \( y = 2x + 3 \)

b) \( y = 2x - 3 \)

c) \( y = 2x + 1 \)

d) \( y = 2x - 1 \)

e) \( y = 2x \)
25. If \((a, b)\) is a point on the ellipse \(4x^2 + y^2 = 4\) which is farthest away from the point \((1, 0)\), then \(b^2 =\)

a) \(\frac{32}{9}\)

b) \(\frac{1}{9}\)

c) 11

d) 9

e) \(\frac{29}{11}\)

26. Let \(x_1 = 1\) and \(x_2 = 1.1\) be the first and second Newton’s approximations of a zero of the differentiable function \(f\). The value of \(\frac{f(1)}{f'(1)}\) is

a) \(-0.1\)

b) 0.1

c) \(-10\)

d) \(-1.1\)

e) 1.1
27. Find the most general antiderivative of the function \( f(x) = \frac{2\cos^2 x - 1}{\cos^2 x} \)

a) \( 2x - \tan x + c \)
b) \( 2x - \sec x \tan x + c \)
c) \( 2x + c \)
d) \( x + c \)
e) \( 2x - \sec x + c \)

28. The greatest area of the rectangle that has its base on the \( x \)-axis and is inscribed in the parabola \( y = 9 - x^2 \) is equal to

a) \( 12\sqrt{3} \)
b) \( 6\sqrt{3} \)
c) \( 9 \)
d) \( 18 \)
e) \( \frac{2}{\sqrt{3}} \)