1. If $\lim\limits_{x\to 0} \dfrac{3^{ax}-1}{x} = 1,$ then the value of a equals

a) $\dfrac{1}{\ln 3}$
b) $\ln 3$
c) 3
d) $-\ln 3$
e) $\dfrac{1}{3}$

2. If $f(x) = \begin{cases} \dfrac{\sin(\cos x - 1)}{x} & \text{if} \quad x \neq 0 \\ a & \text{if} \quad x = 0 \end{cases}$. The value of a which makes the function $f(x)$ continuous every where is

a) 0
b) π
c) $-\pi$
d) -1
e) $\dfrac{\pi}{2}$

3. If $\cosh x = \dfrac{5}{3}$ and $x < 0$, then $3\sinh x - 5\tanh x$ is

a) 0
b) -8
c) 8
d) -6
e) 6

4. If there are two tangent lines from the point $P(0,-1)$ that touch the graph of $f(x) = x^2$ at $x = a$ and $x = b$, then $f'(a) + f'(b) =$

a) 0
b) 2
c) 4
d) 8
e) 16

5. Using the graph of $f(x) = \tan x$, the maximum value of δ such that $|f(x)| < 0.1$ whenever $|x| < \delta$ is equal to

   a) $\tan^{-1} 0.1$
   b) $\tan^{-1}(-0.1)$
   c) $\tan^{-1} 0.2$
   d) $2\tan^{-1} 0.1$
   e) 0

6. If $G(x) = 60\sqrt{x} - 162\sqrt[3]{x}$, then the slope of the tangent line to the graph of $y = G'(x)$ at $x = 1$ is

   a) 21
   b) -18
   c) 61
   d) -13
   e) 28

7. If $h(x) = \dfrac{\sec x}{g(x)}$ with $h'(\pi) = 2$ and $g(\pi) = \sqrt{2}$, then $g'(\pi)$ is

a) 4
b) $4 + \sqrt{2}$
c) $-2\sqrt{2}$
d) 5
e) $2\sqrt{2}$

8. If $f(x) = \sec x$, then $f''(\dfrac{\pi}{4}) =$

a) $3\sqrt{2}$
b) 1
c) $\dfrac{12}{13}$
d) $\dfrac{5}{4}\sqrt{3}$
e) 9

9. The slope of tangent line to the graph of the equation $3x^{\frac{4}{3}} + xy + 3y^{\frac{4}{3}} = 59$ at the point $(1, 8)$ is

   a) $-\frac{4}{3}$
   b) $\frac{19}{8}$
   c) $-\frac{20}{9}$
   d) $\frac{16}{7}$
   e) $\frac{20}{9}$

10. The slope of the tangent line to the graph of $y = (\ln x)^x$ at the point $(e, 1)$ is

   a) 1
   b) 2
   c) e
   d) 0
   e) -1

11. If $f(x) = \ln \left| \dfrac{(x^2+4)^{5/2}}{(x+\sqrt{x})^{3/2}} \right|$, then $f'(1) =$

a) $-\dfrac{1}{8}$

b) $\dfrac{5}{9}$

c) $\dfrac{7}{8}$

d) $-\dfrac{4}{9}$

e) $-\dfrac{13}{8}$

12. A particle is moving along the curve $y = \sqrt{x}$. As the particle passes through the point $(4, 2)$, its x-coordinate increases at a rate of 3 cm/s. The rate of change of the distance from the particle to the origin at that instant is

a) $\dfrac{27}{4\sqrt{5}}$

b) 9

c) $\dfrac{2}{\sqrt{5}}$

d) $\dfrac{2}{3\sqrt{7}}$

e) $\dfrac{4}{\sqrt{11}}$

13. Using differentials or linear approximation the number $\sqrt[3]{26}$ is estimated as

a) $\frac{80}{27}$

b) $\frac{82}{27}$

c) 2.99

d) 3.1

e) $\frac{85}{27}$

14. The value of $\frac{d}{dx}(\sinh^{-1}(\operatorname{csch} x))$ at $x = \ln 3$ is

a) $-\frac{3}{4}$

b) $-\frac{5}{4}$

c) $\frac{7}{6}$

d) $\frac{11}{6}$

e) $\frac{7}{4}$

15. The position function of a particle moving along a straight line is $s(t) = 8t - 3t^2$ for t in $[1, 2]$, where t is measured in seconds and s in meters. The particle is speeding up when

a) $\frac{4}{3} < t < 2$

b) $1 < t < \frac{4}{3}$

c) $1 < t < 2$

d) $1 < t < \frac{3}{2}$

e) $\frac{3}{2} < t < 2$

16. The radius of a circular disk is measured to be 5 cm with a maximum error in measurement of 0.1 cm. Using differentials, the maximum error in calculating circumference of the circular disk is

a) $\frac{\pi}{5}$ cm

b) π cm

c) $\frac{\pi}{10}$ cm

d) $\frac{\pi}{2}$ cm

e) $\frac{\pi}{50}$ cm

17. The sum of the absolute maximum value and the absolute minimum value of the function $f(x) = 2\sin x + \cos 2x$ on the internal $[0, \frac{\pi}{2}]$ is

a) $\dfrac{5}{2}$

b) 2

c) $\dfrac{3}{2}$

d) 3

e) $\dfrac{7}{2}$

18. The sum of all critical numbers of the function $f(x) = \dfrac{(x-4)^2}{\sqrt[3]{x+1}}$ is

a) 2

b) 1

c) 4

d) -2

e) -1

19. If $f(5) = -\frac{5}{2}$ and $f'(x) \geq -\frac{1}{2}$ for $3 \leq x \leq 5$, then the largest possible value of $f(3)$ is

a) $-\frac{3}{2}$

b) $\frac{1}{2}$

c) $-\frac{1}{4}$

d) $-\frac{2}{5}$

e) 0

20. If c is a number satisfying the conclusion of the Mean Value Theorem when applied to $f(x) = \tan^{-1} x$ on $[0, 1]$, then $\pi c^2 =$

a) $4 - \pi$

b) 4

c) $\pi + 1$

d) 2π

e) $\pi - 2$

21. The function $f(x) = \dfrac{x}{x^2+1}$ is increasing on

 a) $(-1, 1)$
 b) $(-\infty, -1) \cup (1, \infty)$
 c) $(-\infty, 0)$
 d) $(0, \infty)$
 e) $(-\infty, \infty)$

22. If the function $f(x) = x^3 + 2ax^2 - 3bx + 1$ has an inflection point at $(1, 2)$, then $2a + b^3$ equals

 a) -4
 b) -2
 c) 2
 d) 3
 e) -1

23. The value of the limit $\lim_{x\to 0}(1-\sin x)^{\frac{1}{x}}$ equals

a) $\frac{1}{e}$

b) e

c) 1

d) $\frac{1}{\sqrt{e}}$

e) 0

24. The slant asymptote of $y = \frac{2x^3+3x^2+20}{x^2+1}$ is

a) $y = 2x+3$

b) $y = 2x-3$

c) $y = 2x+1$

d) $y = 2x-1$

e) $y = 2x$

25. If (a, b) is a point on the ellipse $4x^2 + y^2 = 4$ which is farthest away from the point $(1, 0)$, then $b^2 =$

a) $\frac{32}{9}$

b) $\frac{1}{9}$

c) 11

d) 9

e) $\frac{29}{11}$

26. Let $x_1 = 1$ and $x_2 = 1.1$ be the first and second Newton's approximations of a zero of the differentiable function f. The value of $\dfrac{f(1)}{f'(1)}$ is

a) -0.1

b) 0.1

c) -10

d) -1.1

e) 1.1

27. Find the most general antiderivative of the function $f(x) = \dfrac{2\cos^2 x - 1}{\cos^2 x}$

a) $2x - \tan x + c$
b) $2x - \sec x \tan x + c$
c) $2x + c$
d) $x + c$
e) $2x - \sec x + c$

28. The greatest area of the rectangle that has its base on the x-axis and is inscribed in the parabola $y = 9 - x^2$ is equal to

a) $12\sqrt{3}$
b) $6\sqrt{3}$
c) 9
d) 18
e) $\dfrac{2}{\sqrt{3}}$