Math 132 — Quiz 4 — Date 08/10/2011

Ex1: Use logarithmic differentiation to find y'

a) $y = (x^x)^x$

b) $y = \dfrac{\sqrt{x^2+1}\,\sqrt[3]{x^2+2}}{(2x^3+6x)^{2/5}}$

Ex2: Use implicit differentiation to find y'

$$\ln(xy^2) = xy$$

Solution

Ex1: a) $\ln y = x\ln(x^x) = x^2\ln x$. Thus

$$\frac{y'}{y} = 2x\ln x + x = x(1+2\ln x). \text{ Hence } y' = x(1+2\ln x)(x^x)^x$$

b) $\ln y = \frac{1}{2}\ln(x^2+1) + \frac{1}{3}\ln(x^2+2) - \frac{2}{5}\ln(2x^3+6x)$. Thus

$$\frac{y'}{y} = \frac{x}{x^2+1} + \frac{2x}{3(x^2+2)} - \frac{2}{5}\,\frac{6x^2+6}{2x^3+6x} \quad \text{Hence}$$

$$y' = \left[\frac{x}{x^2+1} + \frac{2x}{3(x^2+2)} - \frac{12}{5}\,\frac{x^2+1}{2x^3+6x}\right]\frac{\sqrt{x^2+1}\,\sqrt[3]{x^2+2}}{(2x^3+6x)^{2/5}}$$

Ex2: $\dfrac{d}{dx}\ln(xy^2) = \dfrac{d}{dx}(xy)$. Thus

$$\frac{d}{dx}\left[\ln x + 2\ln y\right] = \frac{d}{dx}(xy) \quad \text{Now}$$

$$\frac{d}{dx}\left[\ln x + 2\ln y\right] = \frac{1}{x} + 2\frac{y'}{y}$$

and $\dfrac{d}{dx}(xy) = y + xy'$

Therefore

$$\frac{1}{x} + \frac{2y'}{y} = y + xy'$$

$$\Leftrightarrow \quad y'\left(\frac{2}{y} - x\right) = y - \frac{1}{x}$$

$$\Leftrightarrow \quad y' = \frac{y - \frac{1}{x}}{\frac{2}{y} - x} = \frac{yx - 1}{2 - xy} \, \frac{y}{x} = \frac{y(yx-1)}{x(2-xy)}$$