Math 132 Quiz 3 Date 01/10/2011

Ex1: Find the derivative of the following functions.

a) $y = \sqrt[3]{(7-3x^2)^2}$

b) $y = \dfrac{-3}{4(x^5+2x-5)^4}$

Ex2: Find an equation of the tangent line to $y = 5x^3\sqrt{3+2x^4}$ at $x=1$.

Solution.

Ex1: a) $y = \left(7-3x^2\right)^{2/3}$. So

$$\frac{dy}{dx} = \frac{2}{3}\left(-6x\right)\left(7-3x^2\right)^{2/3-1} = \frac{-4x}{\sqrt[3]{7-3x^2}}$$

b) $y = -\frac{3}{4}\left(x^5+2x-5\right)^{-4}$. So

$$\frac{dy}{dx} = \left(-\frac{3}{4}\right)\left(-4\right)\left(5x^4+2\right)\left(x^5+2x-5\right)^{-5} = \frac{3\left(5x^4+2\right)}{\left(x^5+2x-5\right)^5}$$

Ex2: $$\frac{dy}{dx} = 5\left[3x^2\sqrt{3+2x^4} + x^3\frac{1}{2}\left(8x^3\right)\left(3+2x^4\right)^{-1/2}\right]$$

$$= 5\left[3x^2\sqrt{3+2x^4} + \frac{4x^6}{\sqrt{3+2x^4}}\right]$$

The slope of the tangent line is given by:

$$\left.\frac{dy}{dx}\right|_{x=1} = 5\left[3\sqrt{5} + \frac{4}{\sqrt{5}}\right] = 5 \cdot \frac{19}{\sqrt{5}} = 19\sqrt{5}$$

An equation of the tangent line is:

$$y = 19\sqrt{5}\,(x-1) + \underbrace{5\sqrt{5}}_{\text{The image of } 1}$$

$$= 19\sqrt{5}\,x - 14\sqrt{5} = \sqrt{5}\,(19x - 14)$$