Math 132 Quiz 5 17/10/2011

Ex1. Let $e^x - e^y = x^2 + y^2$. Find y''.

Ex2. Let $f(x) = x^{5/3} + 5x^{2/3}$

a) Find all critical points of f

b) Find intervals on which f is increasing or decreasing.

c) Find all absolute and relative extrema of f.

Solution:

Ex1. We differentiate both sides:

$$\frac{d}{dx}(e^x - e^y) = \frac{d}{dx}(x^2 + y^2). \text{ Then:}$$

$$e^x - y'e^y = 2x + 2y'y. \text{ We solve it for } y'\text{:}$$

$$y'(2y + e^y) = e^x - 2x \quad \text{i.e} \quad y' = \frac{e^x - 2x}{e^y + 2y}$$

Now we differentiate again:

$$y'' = \frac{(e^x - 2)(e^y + 2y) - (e^x - 2x)(y'e^y + 2y')}{(e^y + 2y)^2}$$

$$= \frac{(e^x - 2)(e^y + 2y) - y'(e^x - 2x)(e^y + 2)}{(e^y + 2y)^2}$$

$$= \frac{(e^x - 2)(e^y + 2y)^2 - (e^x - 2x)^2(e^y + 2)}{(e^y + 2y)^3}$$

Ex2: $D_f = \mathbb{R}$

a) $f'(x) = \frac{5}{3} x^{2/3} + \frac{10}{3} x^{-1/3} = \frac{5}{3} x^{2/3}\left(1 + \frac{2}{x}\right)$

Thus $f'(x) = 0$ iff $x = -2$. Hence f has two critical points which are:

$-2 \leftarrow$ where the derivative is zero

$0 \leftarrow$ where the derivative does not exist

b)

x	$-\infty$		-2		0		$+\infty$
$f'(x)$		$+$	|	$-$	‖	$+$	
$f(x)$	$-\infty$	$\nearrow$	$f(-2)$	$\searrow$	0	$\nearrow$	$+\infty$

f is increasing on $(-\infty, -2) \cup (0, +\infty)$

f is decreasing on $(-2, 0)$

c) -2 is a relative maximum

0 is a relative minimum