MATH 201.1 (Term 111)

Quiz 4 (Sects. 14.1-2) Duration: 15mn

Name: **ID number:**

1.)(5pts)What are the domain and range of the function $f(x,y) = \frac{1+\sqrt{4-x^2-y^2}}{e^{x^2+y^2}}$.

2.)(5pts)Is the function

$$h(x,y) = \begin{cases} \frac{x^3+y^3}{x^2+y^2}, & \text{when } (x,y) \neq (0,0) \\ 0 & \text{when } (x,y) = (0,0), \end{cases}$$

continuous at $(0,0)$**? Justify your answer.**

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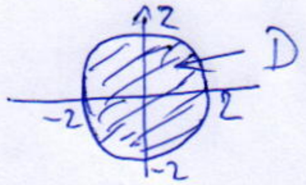
1) $4-x^2-y^2 \geq 0$

$x^2+y^2 \leq 4$

The domain of f is all the points (x,y) inside the disk of center $(0,0)$ and radius 2.



For $(x,y) \in D$, we have

$0 \leq x^2+y^2 \leq 4$

$-4 \leq -x^2-y^2 \leq 0$

$0 \leq 4-x^2-y^2 \leq 4$

$0 \leq \sqrt{4-x^2-y^2} \leq 2$

$1 \leq 1+\sqrt{4-x^2-y^2} \leq 3$ ①

On the other hand, we have

$1 \leq e^{x^2+y^2} \leq e^4$

$\frac{1}{e^4} \leq \frac{1}{e^{x^2+y^2}} \leq 1$ ②

We multiply the inequalities ① and ②. We find

$\frac{1}{e^4} \leq \frac{1+\sqrt{4-x^2-y^2}}{e^{x^2+y^2}} \leq 3$

Range $f = [e^{-4}, 3]$

2) We have

$\left|\frac{x^3+y^3}{x^2+y^2}\right| \leq |x| + |y| \leq 2\sqrt{x^2+y^2} \leq 2\delta$

For any $\varepsilon > 0$, we can choose $2\delta = \varepsilon$, that is, $\delta = \varepsilon/2$

This implies

$\left|\frac{x^3+y^3}{x^2+y^2}\right| < \varepsilon$ whenever $\sqrt{x^2+y^2} < \delta$

$\Rightarrow \lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2+y^2} = 0$

So, h is continuous at $(0,0)$ since $h(0,0)=0$.

MATH 201.5 (Term 111)

Quiz 4 (Sects. 14.1-2) Duration: 15mn

Name: **ID number:**

1.)(5pts)What are the domain and range of the function $f(x,y) = \frac{1+x^2+y^2}{1+\sqrt{1-x^2-y^2}}$.

2.)(5pts)Is the function

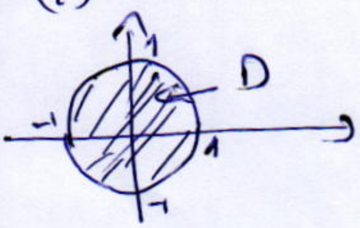
$$g(x,y) = \begin{cases} \frac{x^2y-xy^2}{x^3+y^3}, & \text{when } (x,y) \neq (0,0) \\ 1 & \text{when } (x,y) = (0,0), \end{cases}$$

continuous at $(0,0)$? **Justify your answer.**

1.) $1-x^2-y^2 \geq 0$

$x^2+y^2 \leq 1$

The domain of f is all the points (x,y) inside the disk of centre $(0,0)$ and radius 1



For any $(x,y) \in D$,

$0 \leq x^2+y^2 \leq 1$

$-1 \leq -x^2-y^2 \leq 0$

$0 \leq 1-x^2-y^2 \leq 1$

$0 \leq \sqrt{1-x^2-y^2} \leq 1$

$1 \leq 1+\sqrt{1-x^2-y^2} \leq 2$

$\frac{1}{2} \leq \frac{1}{1+\sqrt{1-x^2-y^2}} \leq 1$ (1)

On the other hand, we have

$1 \leq 1+x^2+y^2 \leq 2$ (2)

We multiply (1) and (2).

We find

$\frac{1}{2} \leq \frac{1+x^2+y^2}{1+\sqrt{1-x^2-y^2}} \leq 2$

Range $f = [1/2, 2]$

2.) $\lim_{(x,0)\to(0,0)} g(x,y) = 0$

$\lim_{(x,2x)\to(0,0)} g(x,y) = -\frac{2}{9}$

So, the limit of g at $(0,0)$ does not exist.

$\Rightarrow$ g is not continuous at $(0,0)$.