MATH 201.1 (Term 111)

Quiz 5 (Sects. 14.8, 15.1-3) Duration: 15mn

Name: **ID number:**

1.)(10pts)Use Lagrange multipliers to find the maximum and minimum values of the function subject to the given constraint

$$f(x,y,z) = 2x + 6y + 10z; \quad x^2 + y^2 + z^2 = 35.$$

2.)(10pts)Evaluate the integral by reversing the order of integration

$$\int_0^1 \int_{\sqrt{y}}^1 \sqrt{2x^3+3}\, dx\, dy.$$

---

1) We solve $\begin{cases} \nabla f(x,y,z) = \lambda \nabla g(x,y,z) \\ x^2+y^2+z^2=35 \end{cases}$

$\nabla f(x,y,z) = \langle 2, 6, 10 \rangle$

$\nabla g(x,y,z) = \langle 2x, 2y, 2z \rangle$

$\begin{cases} 2 = 2\lambda x & \rightarrow x = \frac{1}{\lambda} \\ 6 = 2\lambda y & \rightarrow y = \frac{3}{\lambda} \\ 10 = 2\lambda z & \rightarrow z = \frac{5}{\lambda} \\ x^2+y^2+z^2=35 \end{cases}$

$\left(\frac{1}{\lambda}\right)^2 + \left(\frac{3}{\lambda}\right)^2 + \left(\frac{5}{\lambda}\right)^2 = 35$

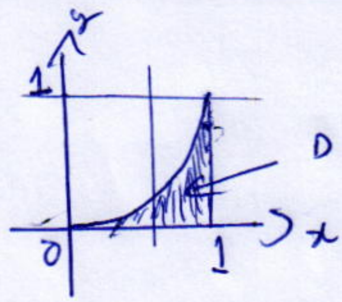
$1 + 9 + 25 = 35\lambda^2$

$\lambda^2 = 1 \Rightarrow \lambda = \pm 1$

$f(-1,-3,-5) = -70$ minimum

$f(1,3,5) = 70$ maximum

2)



if we fix $0 \le x \le 1$, then $0 \le y \le x^2$

$\Rightarrow \int_0^1 \int_0^{x^2} \sqrt{2x^3+3}\, dy\, dx$

$= \int_0^1 x^2 \sqrt{2x^3+3}\, dx$

$= \left[\frac{1}{9}\left(2x^3+3\right)^{3/2}\right]_0^1$

$= \frac{1}{9}\left(5^{3/2} - 3^{3/2}\right)$

MATH 201.5 (Term 111)

Quiz 5 (Sects. 14.8, 15.1-3) Duration: 15mn

Name: ID number:

1.)(10pts)Use Lagrange multipliers to find the maximum and minimum values of the function f subject to the given constraint

$$f(x, y, z) = 8x - 4z, \quad x^2 + 10y^2 + z^2 = 5.$$

2.)(10pts)Evaluate the integral by reversing the order of integration

$$\int_0^1 \int_{x^2}^1 x^3 \sin(2y^3)\, dy\, dx.$$

---

1)
$$\begin{cases} \nabla f(x,y,z) = \lambda \nabla g(x,y,z) \\ x^2 + 10y^2 + z^2 = 5 \end{cases}$$

$\nabla f(x,y,z) = \langle 8, 0, -4 \rangle$

$\nabla g(x,y,z) = \langle 2x, 20y, 2z \rangle$

$$\begin{cases} 8 = 2\lambda x & (1) \\ 0 = 20\lambda y & (2) \\ -4 = 2\lambda z & (3) \\ x^2 + 10y^2 + z^2 = 5 & (4) \end{cases}$$

From (2), we have $\lambda = 0$ or $y = 0$

Now, we can see that $\lambda = 0$ is impossible due to (1) and (3).

So, $y = 0$ and $x = \frac{4}{\lambda}$, $z = \frac{-2}{\lambda}$

$\Rightarrow \left(\frac{4}{\lambda}\right)^2 + \left(\frac{-2}{\lambda}\right)^2 = 5$

$16 + 4 = 5\lambda^2$

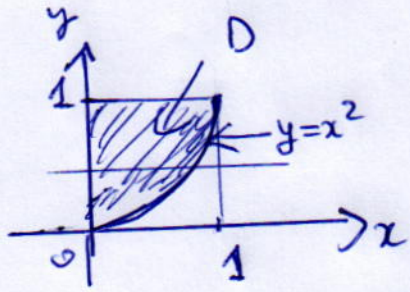
$20 = 5\lambda^2$

$\lambda^2 = 4 \Rightarrow \underline{\lambda = \pm 2}$

$f(-2, 0, 1) = -20$ Minimum

$f(2, 0, -1) = 20$ Maximum

2)



if we fix $0 \le y \le 1$, then $0 \le x \le \sqrt{y}$

$$\int_0^1 \int_0^{\sqrt{y}} x^3 \sin(2y^3)\, dx\, dy$$

$$= \frac{1}{4} \int_0^1 \left[x^4\right]_0^{\sqrt{y}} \sin(2y^3)\, dy$$

$$= \frac{1}{4} \int_0^1 y^2 \sin(2y^3)\, dy$$

$$= \left[-\frac{1}{24} \cos(2y^3)\right]_0^1$$

$$= \frac{1 - \cos 2}{24}$$