MATH 201.1 (Term 111)

Quiz 6 (Sects. 15.4-6) Duration: 20mn

Name: ID number:

1.)(5pts)Evaluate $\int_0^3 \int_0^{\sqrt{9-x^2}} e^{x^2+y^2}\, dy dx$.

2.)(5pts) Find the volume of the solid F outside the cylinder $x^2+y^2=1$ and inside the hemisphere $x^2+y^2+z^2=4$, $z \geq 0$.

3.)(5pts)Express a an iterated integral $\iiint_E f(x,y,z)dV$, where E is the solid bounded by the planes $z=0$, $x=0$, $y=1$ and $z=y-x$.

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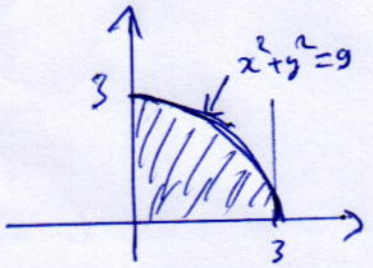
1) $D=\{(x,y) / 0 \leq x \leq 3,\ 0 \leq y \leq \sqrt{9-x^2}\}$



We convert into polar coordinates

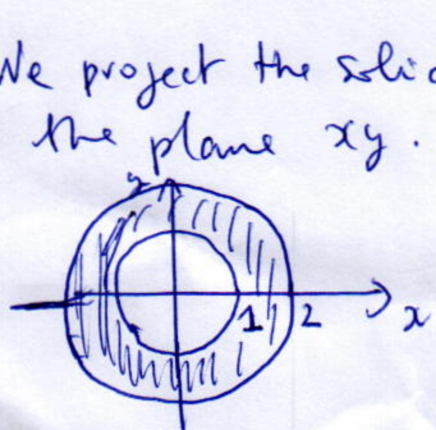
$D=\{(r,\theta) / 0 \leq \theta \leq \frac{\pi}{2},\ 0 \leq r \leq 3\}$

$$I = \int_0^{\pi/2} \int_0^3 e^{r^2}\, r\, dr\, d\theta$$

$$= \int_0^{\pi/2} \left[\frac{e^{r^2}}{2}\right]_0^3 d\theta$$

$$I = \left(\frac{e^9-1}{2}\right)\frac{\pi}{2}$$

2) We project the solid into the plane xy.



$V = \iint_D \sqrt{4-x^2-y^2}\, dA$, where

$D=\{(x,y)\ \ 1 \leq x^2+y^2 \leq 4\}$

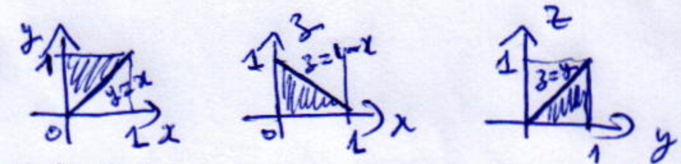
Now, we convert into polar coordinates $D=\{(r,\theta)/ 0 \leq \theta \leq 2\pi,\ 1 \leq r \leq 2\}$

$$V = \int_0^{2\pi} \int_1^2 \sqrt{4-r^2}\, r\, dr\, d\theta$$

$$= \int_0^{2\pi} \left[-\frac{1}{3}(4-r^2)^{3/2}\right]_1^2 d\theta$$

$$= 2\pi\sqrt{3}$$

3.)



$$I_1 = \int_0^1 \int_x^1 \int_0^{y-x} f(x,y,z)\, dz\, dy\, dx$$

$$I_2 = \int_0^1 \int_0^y \int_0^{y-x} f(x,y,z)\, dz\, dx\, dy$$

$$I_3 = \int_0^1 \int_0^{1-x} \int_{z+x}^1 f(x,y,z)\, dy\, dz\, dx \qquad I_5 = \int_0^1 \int_0^y \int_0^{y-z} f(x,y,z)\, dx\, dz\, dy$$

$$I_4 = \int_0^1 \int_0^{1-z} \int_{z+x}^1 f(x,y,z)\, dy\, dx\, dz \qquad I_6 = \int_0^1 \int_z^1 \int_0^{y-z} f(x,y,z)\, dx\, dy\, dz$$

MATH 201.5 (Term 111)

Quiz 6 (Sects. 15.4-6) Duration: 20mn

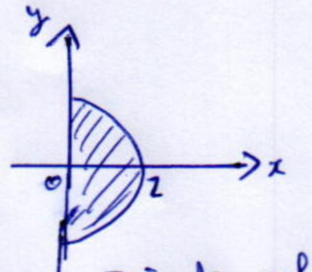
Name: ID number:

1.)(5pts)Evaluate $\iint_E \sqrt{4-x^2-y^2}dA$, where $E=\{(x,y)/0\le x^2+y^2\le 4,\ x\ge 0\}$.

2.)(5pts)Find the volume of the solid G bounded by $z=1$ and $z=\sqrt{x^2+y^2}$.

3.)(5pts)Reverse tho order of the triple integral $\int_0^1\int_0^{x^2}\int_0^y f(x,y,z)dzdydx$ in the form $dydxdz$ and $dxdzdy$

1.)



We convert E into polar coordinates

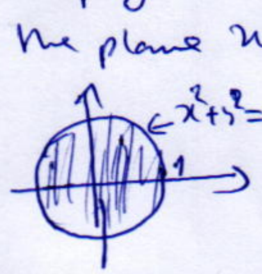
$E=\{(r,\theta)/ -\frac{\pi}{2}\le\theta\le\frac{\pi}{2},\quad 0\le r\le 2\}$

$I=\int_{-\pi/2}^{\pi/2}\int_0^2 \sqrt{4-r^2}\, r\, dr\, d\theta$

$=\int_{-\frac{\pi}{2}}^{\pi/2}\left[-\frac{1}{3}(4-r^2)^{3/2}\right]_0^2 d\theta$

$=\frac{8}{3}\pi$

2.) We project the solid in the plane xy



$V=\iint_D f(x,y)dA,\quad f(x,y)=1-\sqrt{x^2+y^2}$

$D=\{(x,y)/\ 0\le x^2+y^2\le 1\}$

Now, we convert into polar coordinates.

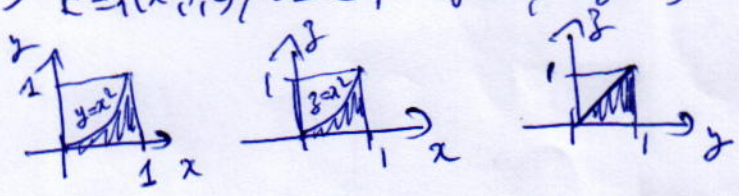
$D=\{(r,\theta)/\ 0\le\theta\le 2\pi,\quad 0\le r\le 1\}$

$V=\int_0^{2\pi}\int_0^1 (1-\sqrt{r^2})\, r\, dr\, d\theta$

$=\int_0^{2\pi}\int_0^1 (r-r^2)\, dr\, d\theta=\int_0^{2\pi}\left[\frac{r^2}{2}-\frac{r^3}{3}\right]_0^1 d\theta$

$=\frac{2\pi}{6}=\pi/3.$

3) $E=\{(x,y,z)/\ 0\le x\le 1,\ 0\le y\le x^2,\ 0\le z\le y\}$



$I=\int_0^1\int_{\sqrt{y}}^1\int_0^y f(x,y,z)\, dz\, dx\, dy$

$=\int_0^1\int_0^{x^2}\int_z^{x^2} f(x,y,z)\, dy\, dz\, dx$

$=\int_0^1\int_{\sqrt{z}}^1\int_z^{x^2} f(x,y,z)\, dy\, dx\, dz$

$=\int_0^1\int_0^y\int_{\sqrt{y}}^1 f(x,y,z)\, dx\, dz\, dy$

$=\int_0^1\int_z^1\int_{\sqrt{y}}^1 f(x,y,z)\, dx\, dy\, dz$