Q1. Find the extreme values of $f(x, y) = xy^2$ on the ellipse $2x^2 + y^2 = 6$ by using the Lagrange multipliers. (10 pts)
Q2. (i) Find the Cartesian Equation of the parametric curve $x = 2 \cos 2t, \ y = 3 \sin 2t; -\pi/4 \leq t \leq \pi/4$. 

(ii) Sketch the above curve in the $xy$-plane. Indicate with an arrow the direction in which the curve is traced as the parameter $t$ increases.

(iii) Set up the integral that represents the surface area of the solid obtained by revolving the above curve about the y-axis.
Q3. If $f(x, y, z) = \ln(xy^2), x = \cos t\theta, y = \ln(t^2 + \theta), z = \theta^2$, find $\frac{\partial f}{\partial t}$ at $(t, \theta) = (1, \pi)$. (8 pts)

Q4. Find the equation of the plane that contains the points $(1, 2, -3), (3, 1, 0), \text{and} (2, 3, -7)$. (8 pts)
Q5. Consider the function \( f(x, y) = x^2 + y^2 + x^2 y + 4 \).

(i) Show that \((0,0), (\sqrt{2}, -1)\) and \((-\sqrt{2}, -1)\) are the critical points of \( f(x,y) \). (4 pts)

(ii) Find the local minimum and maximum value(s) and the saddle point(s) of \( f(x,y) \), if any. (6 pts)
Q6. Find the value of the integral \( \iiint_E xy \, dV \) where \( E \) is bounded by the parabolic cylinders \( x = y^2 \) and \( y = x^2 \), and the planes \( z = 0 \) and \( z = x + y \). (10pts)
Q7. A solid lies within both the cylinder \( x^2 + y^2 = 1 \) and the paraboloid \( z = 4 - x^2 - y^2 \), and above the \( xy \)-plane. Only **set up** the integral in **cylindrical coordinates** that represents the volume of the solid. (8 pts)
Q8. Evaluate the integral \[
\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{0}^{\sqrt{1-x^2} - \sqrt{1-x^2-y^2}} z\sqrt{x^2 + y^2 + z^2} \, dz \, dy \, dx.
\] (10 pts)
Part II (8 MCQ: 8 pts/each)  Encircle your Choice for each MCQ on the front page of your answer book

Q1. If $A$ is the area inside the circle $r = 3\cos \theta$ and outside the cardioid $r = 1 + \cos \theta$, then $2A =$

(a) $\frac{3\pi}{2}$
(b) $3\pi$
(c) $2\pi$
(d) $\pi$
(e) $\frac{\pi}{2}$

Q2. If $|\vec{a}|,|\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 2$, then $|\vec{a} \times \vec{b}|^2 =$

(a) 25
(b) 15
(c) 8/3
(d) $25/3$
(e) 5
Q3. The \[ \lim_{{(x,y) \to (0,0)}} \frac{3(x^2 + y^2)}{\sqrt{x^2 + y^2 + 1} - 1} \]

(a) \(= -\frac{1}{2}\) 
(b) \(= 6\) 
(c) \(= 0\) 
(d) \(= 4\) 
(e) does not exist

Q4. If \(ax + by + cz + d = 0\) is the equation of the tangent plane to the surface \(z = \sqrt{9 - x^2 - y^2}\) at \((2, -2, 1)\), then \(\frac{a + b + c + d}{2}\) equals

(a) \(-4\) 
(b) \(\frac{7}{2}\) 
(c) \(-5\) 
(d) \(-16\) 
(e) \(0\)
Q5. If the rate of change of $f(x,y) = xe^y$ at the point $P(2,0)$ in the direction from $P$ to $Q(1/2, 2)$ is $V$, then $2V =$

(a) $2$
(b) $2\sqrt{2}$
(c) $-3/5$
(d) $5 e^2$
(e) $4$

Q6. If $D$ is a triangular region with vertices $(0, 0)$, $(0, 2)$ and $(2, 2)$, then the value of the integral $\iint_D 2xy dA$ is

(a) $2$
(b) $12$
(c) $8$
(d) $1$
(e) $4$
Q7. The value of the integral \( \int_0^1 \int_0^{\sqrt{y}} 6e^{3x^2} \, dx \, dy \) is

(a) \( e \cdot (e - 1) \)
(b) \( e / 2 \)
(c) \( 2 \cdot (e - 1) \)
(d) \( e - 2 \)
(e) \( 3 \cdot (e - 1) \)

Q8. The iterated integral \( \int_0^{2\sqrt{e-x^2}} \int_0^x \frac{x^2}{1+x^2+y^2} \, dy \, dx \) when changed to polar coordinates can be expressed as

(a) \( \int_0^{\pi/2} \int_0^{2 \cos \theta} \frac{r^3 \cos^2 \theta}{1+r^2} \, dr \, d\theta \)
(b) \( \int_0^{\pi/2} \int_0^{2 \cos \theta} \frac{r^4 \cos^3 \theta}{1+r^2} \, dr \, d\theta \)
(c) \( \int_0^{\pi/2} \int_0^{4 \cos \theta} \frac{r^2 \cos^2 \theta}{1+r^2} \, r \, dr \, d\theta \)
(d) \( \int_0^{\pi/2} \int_0^{\cos \theta} \frac{\cos^2 \theta}{1+r^2} \, r \, dr \, d\theta \)
(e) \( \int_0^{\pi/2} \int_0^{2 \cos \theta} \frac{r^3 \cos^2 \theta}{1+r^2} \, dr \, d\theta \)