Q: 1 Consider the differential equation

\[ y'' + 3y = -18e^{3x} \]  \hspace{1cm} (1)

(a) (5 points) Verify that \( y_1 = \cos \sqrt{3}x \) and \( y_2 = \sin \sqrt{3}x \) are solutions of \( y'' + 3y = 0 \).

(b) (5 points) Find a particular solution of the differential equation (1).

(c) (2 points) Write the general solution of the differential equation (1).
Q:2  (16 points) Find the general solution of the differential equation

\[ xy'' - y' + 4x^3 y = 0 \]

given that \( y_1 = \sin(x^2) \) is a solution.
Q: 3  (16 points) Solve the initial value problem

\[ y''' + y'' - y' - y = 0; \quad y(0) = 0, \quad y'(0) = 0, \quad y''(0) = 4. \]
Q:4  (a) (8 points) Find a linear differential operator that annihilates the function

\[ 9x^3 + x^2(1 - 3x)e^{2x} + 5xe^{-2x} \cos 3x \]

(b) (16 points) Solve the differential equation \( y'' - 5y' + 6y = 3e^x + 2 \sin x \) using undetermined coefficients.
Q:5  (a) (12 points) Use variation of parameters method to find a particular solution of

\[(x^2 - 1)y'' - 2xy' + 2y = (x^2 - 1)^2\]

given that a complementary function is \(y_c = c_1 x + c_2(1 + x^2)\).

(b) (4 points) Write the general solution and find values of \(c_1\) and \(c_2\) using the boundary conditions \(y(0) = 0\) and \(y(1) = 1\).
Q:6 (16 points) Find the general solution of

\[ 4x^2y'' + 8xy' + y = 4x^2 \]