1. Write clearly.
2. Show all your steps.
3. No credit will be given to wrong steps.
4. Do not do messy work.
5. Calculators and mobile phones are NOT allowed in this exam.
6. Turn off your mobile.
1. Find the particular solution of the separable DE

\[
\frac{dy}{dx} = \frac{xy + 3y - x - 3}{xy - 2y + x - 2}, \quad y(0) = 2
\]
2. Find the general solution of the first order linear DE:

\[(1 + x^2) \frac{dy}{dx} - 2xy = (1 + x^2) \tan^{-1}(x)\]
3. Find a suitable substitution that transforms the DE

\[ ydx + x(\ln x - \ln y - 1)dy = 0. \]

into a separable DE. Find the new equation (Do not solve it).

4. Write the Bernoulli DE as a first order linear DE and do not solve it

\[ \frac{dy}{dx} = \frac{x^2 \ln(x)}{y} - y \]
5. Verify that \( xe^{2y} - \sin(xy) + y^2 = c \) is an implicit solution of the given DE
\[
[e^{2y} - y \cos(xy)]dx + [2xe^{2y} - x \cos(xy) + 2y]dy = 0
\]

6. Use a suitable substitution to reduce the second order DE
\[2yy'' + 5(y')^2 = 7y(y')^3\]
into Bernoulli DE. (Find the new equation but DO NOT solve it)
7. Show that the given DE is exact

\[(2y \sin(x) \cos(x) + y^2 \sin(x))dx + (\sin^2(x) - 2y \cos(x))dy = 0\]

and find the general solution.
8. The population of a town grows at a rate proportional to the population present at time $t$. The initial population of 2000 increases by 20% in 50 years. What will be the population in 100 years?
9. Consider the following homogeneous linear system

\begin{align*}
4x_1 + x_3 - x_5 - 10x_6 &= 0 \\
2x_2 - 3x_3 - 2x_4 - 10x_5 + 6x_6 &= 0 \\
x_1 + 2x_2 - 2x_4 - 2x_5 - 2x_6 &= 0
\end{align*}

(a) Write the system in matrix form.

(b) Use Gauss-Jordan method to solve the given system.