Name:............................................................ID#

1. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x, y) = \phi(x^2 + y^2)$, where $\phi$ is of class $C^{(2)}$, increasing and concave. Show that $f$ is convex on the circular disk $x^2 + y^2 \leq a^2$ if and only if $\phi'(u) + 2u\phi''(u) \geq 0$ whenever $0 \leq u \leq a^2$.

2. Suppose that $f$ satisfies the partial differential equation $\frac{\partial f}{\partial y} = \frac{\partial^2 f}{\partial x^2} + bf$, where $b$ is a scalar. Let $F(x, y) = \exp(-by)f(x, y)$. Show that $\frac{\partial F}{\partial y} = \frac{\partial^2 F}{\partial x^2}$.

3. Let $g : \mathbb{R}^n \to \mathbb{R}^n$ be of class $C^{(1)}$ and suppose that there is a number $c > 0$ such that $\|g(t) - g(s)\| \geq c\|t - s\|$ for all $t, s \in \mathbb{R}^n$. Show that
   (i) $g$ is univalent,
   (ii) $Jg(t) \neq 0$ for all $t \in \mathbb{R}^n$.

4. Let $g : \mathbb{R}^3 \to \mathbb{R}^3$ be given by $g(s, t, u) = (u \cos st) e_1 + (u \sin st) e_2 + (s + u) e_3$.
   Find $Jg(s, t, u)$. If $g$ is univalent find $g^{-1}$.

5. Solve the following system
   \[
   \begin{cases}
   y_1 - 3y_2 + \sin x = 1 \\
   4y_1 + \cos x + 3x = 4
   \end{cases}
   \]